DOA Estimation for Heterogeneous Wideband Sources Based on Adaptive Space-Frequency Joint Processing

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Abstract—For direction-of-arrival (DOA) estimation of heterogeneous wideband sources, we propose a new adaptive space-frequency joint processing algorithm. The algorithm is implemented in the sparse Bayesian learning (SBL) DOA framework, which is named as ASF-SBL algorithm in this paper. The traditional SBL-based DOA methods suffer from the problem of erroneous DOA estimation due to the structural mismatch between the invariant prior and the sparse coefficients. To solve this problem, the ASF-SBL algorithm employs a new space-frequency correlation prior model that can be adaptively changed to fit heterogeneous DOA scenarios. Specifically, nine alternative space-frequency structural patterns are constructed to represent the joint space-frequency characteristics of spatial sparse signals. By evaluating the space-frequency correlation of the sparse coefficients updated in each iteration under SBL framework, a suitable pattern is selected from the nine choices to determine the adaptive prior of each coefficient. This adaptive method leads to accurate DOA estimation in different wideband sources scenarios.

In addition, we introduce a distributed processing method to extend the ASF-SBL algorithm to two-dimensional DOA estimation. This extension is achieved by decoupling the DOA estimation into two one-dimensional estimations. The decoupling avoids the problems of a huge redundant dictionary and excessive computational complexity caused by the combination of azimuth and elevation angles. Numerical simulations show that the ASF-SBL algorithm is superior to existing algorithms in DOA estimation of heterogeneous sources.

Index Terms—Direction-of-arrival, sparse Bayesian learning, space-frequency joint processing, adaptive structural pattern prior, ASF-SBL algorithm.

NOMENCLATURE

\( (\cdot)^H \) The Hermitian transpose of a matrix.
\( (\cdot)^T \) The transpose of a matrix.
\( (\cdot)^* \) The complex conjugate of a matrix.
\( (\cdot) \) The estimation result of a variable.
\( \text{Tr}(\cdot) \) The trace of a matrix.
\{\( \theta_k, \phi_k \)\} The DOA of the \( k \)-th source.
\{\( \alpha_k, \beta_k \)\} The space angle of the \( k \)-th source.
\( \Theta^{(\theta)} \) The angle set \( \Theta^{(\theta)} = [\theta_1, \ldots, \theta_L] \) obtained by dividing the entire airspace angle range according to a pre-defined grid.
\( X \) The sparse coefficient matrix, also called space frequency matrix. \( x_f \) and \( X_{l,f} \) are the \( f \)-th column and the \( (l, f) \)-th element of the matrix, respectively.
\( \bar{X}_{l,f} \) Matrix \( \bar{X}_{l,f} = [X_{l-1,f}, X_{l,f}, X_{l+1,f}] \).
\( \Phi_f \) The over-complete dictionary, and its \( \theta_{l,k} \)-th column is \( \Phi_f(\theta_{l,k}) \).
\( \Phi_f(\hat{\theta}_{l,k}) \) The first-order derivative of \( \Phi_f(\hat{\theta}_{l,k}) \) with respect to \( \theta_{l,k} \).
\( S \) The support matrix of the space coefficient matrix \( X \). \( S_f \) and \( S_{l,f} \) are the \( f \)-th column and the \( (l, f) \)-th element of the matrix, respectively.
\( \mathbf{s}_f \) The diagonal matrix \( \mathbf{s}_f = \text{diag}(S_f) \).
\( V \) The hyperparameter matrix corresponding to \( X \), and its \( (l, f) \)-th element and \( f \)-th column are \( V_{l,f} \) and \( \nu_f \), respectively.
\( \nu_v \) The correlation parameter matrices in the spatial domain and the frequency domain, respectively.
\( \gamma \) The probability matrix, and its \( (l, f) \)-th element \( \gamma_{l,f} \) is the probability of \( S_{l,f} = 1 \).
\( \delta \) The off-grid deviation vector \( \delta = [\delta_1, \ldots, \delta_L]^T \), and \( \Delta = \text{diag}(\delta) \).

I. INTRODUCTION

Problems associated with the direction-of-arrival (DOA) estimation of wideband sources is a key issue in array signal processing and many solutions have been reported [1]–[10]. Most of these solutions are implemented based on the assumption that the frequency characteristics of the sources are consistent. However, in practical applications [11], [12] such as UAV and helicopter direction-finding, the fundamental
frequency and harmonic frequency of each target is different. This difference can result in scenarios where the frequency bands of the wideband sources completely overlap, partially overlap, or non-overlap. Variations in the degree of overlap can lead to inaccurate DOA estimations if it assumed that multiple sources share the same frequency band.

The incoherent signal-subspace method (ISM) [1], [2] and the coherent signal-subspace method (CSM) [3], [6]–[8] are the most commonly used traditional wideband DOA estimation algorithms. ISMs average the results of the subspace method at each frequency point to achieve DOA estimation. CSMs use a focusing matrix to convert the wideband problem into a narrowband problem and can solve the defect of ISMs, which cannot be used for coherent sources. The acquisition of the focusing matrix is based on the initial value of the DOA. Any error on the initial value will cause a focusing error and seriously affect the CSM’s performance. The focusing error can be avoided by the use of the test of orthogonality of projected subspace (TOPS) method [13], [14] and its extension method [15], which does not require beamforming to obtain the initial value of the DOA. These algorithms can be applied to the DOA estimation of wideband multi-sources sharing the same frequency band. However, they cannot adapt to situations with low signal-noise-ratio (SNR), small snapshots and an unknown number of sources.

The sparse representation methods [16]–[24] based on the spatial sparsity of sources can realize the DOA estimation in the case of low SNR and small snapshots, and do not need to pre-estimate the number of sources. The Block FOCal Undetermined System Solver (BFOCUSS) algorithm [18], subband information fusion (SIF) algorithm [19], [20], L1-Singular Value Decomposition (L1-SVD) [21], Orthogonal Matching Pursuit (OMP) [22] and its extended Distributed Compressive Sensing-Simultaneous OMP (DCS-SOMP) [23] all combine the whole frequency information of the received data, and convert the wideband DOA problem into a multi-dictionary joint optimization problem to achieve accurate DOA estimation. Sparse Bayesian learning (SBL) algorithms can flexibly provide different priors based on signal characteristics, and solve sparse solutions in a probabilistic manner [24]–[31]. Wideband DOA estimation based on SBL framework is, therefore, very popular and algorithms such as the Extended Block-SBL (EBSBL) algorithm [32], [33], cluster algorithm [34], Pattern coupled-SBL (PC-SBL) algorithm [35], [36] and Hybrid clustered structure sparse reconstruction algorithm [37] have appeared to solve the block sparse signal recovery problem with unknown structures.

However, these algorithms assume that all sources share the same frequency band and are implemented by joint processing of the frequency domain data. The application of these algorithms to scenarios containing wideband sources with different frequency characteristics will, therefore, yield incorrect DOA estimations.

Ref. [38] presents a potential solution for the DOA estimation of wideband sources with different frequency characteristics. In this method, the clustering characteristics of the Dirichlet process prior under SBL framework are used to cluster the measurement values with the same sparse pattern into different subbands. Joint sparsity is then applied to each cluster to obtain DOA estimation and suppress spatial aliasing. The method only focuses on the correlation between frequencies in the same cluster and does not consider the spatial correlation of sources. This negligence will affect the spatial resolution, which can cause DOA estimation errors when sources are closely-spaced.

In this paper, we propose a new space-frequency joint processing algorithm under SBL DOA framework, namely ASF-SBL, to solve the DOA estimation for heterogeneous wideband sources with different frequency characteristics. In the ASF-SBL algorithm, we introduce the spatial-frequency statistical correlation of sparse signals and perform joint processing in the spatial-frequency domain for wideband signals. Such processing can reduce spatial aliasing and improve the angular resolution. Unlike the traditional SBL algorithms with an invariant prior model, the ASF-SBL algorithm provides a new space-frequency correlation structure prior model that can be adaptively changed to suit different wideband DOA scenarios. Since the spatial and frequency domain structural patterns can be categorised as strong reception, weak reception or strong rejection, nine space-frequency structural patterns are constructed for the adaptive prior model. Based on the space-frequency correlation of each sparse coefficient, a suitable structural pattern can be selected from the nine choices to determine the prior of the current sparse coefficient. The ASF-SBL algorithm performs variational Bayesian inference (VBI) under the SBL framework to achieve on-grid and off-grid DOA estimation. The solved sparse signal is presented as a space-frequency map to visualize the DOA, frequency characteristics, and amplitude of each source. In addition, we extend the ASF-SBL algorithm to two-dimensional DOA estimation. This extension is achieved by decoupling the DOA estimation into two one-dimensional estimations for distributed processing. Numerical simulations are used to compare the proposed algorithm with other methods used for heterogeneous wideband sources DOA estimation. An emphasis is placed on the algorithm’s performance in situations with low SNR and closely-spaced sources.

This paper is organized as follows. Section II formulates the mathematical model. Section III presents the adaptive structural pattern prior selection process, and proposes a new space-frequency joint processing algorithm for on-grid and off-grid problems. Section IV extends the proposed algorithm to the 2-D DOA estimation. A qualitative discussion of the algorithm is given in Sections V and VI shows the numerical simulations. Conclusions are drawn in Section VII.

II. PROBLEM FORMULATION

Assume that $K$ far-field, wideband, stationary sources with $\theta_k$, $k = 1, \ldots, K$, impinge upon a $M$-elements uniform line array (ULA) with an array element interval of $d$. The discrete Fourier transform (DFT) of the data within a certain observation time gives the frequency domain data of $F$ frequency points (the frequency set is $F = \{1, \ldots, F\}$) [14]–[16], [24], [29]:

$$y_f = A_f z_f + n_f, f = 1, \ldots, F,$$  \hspace{1cm} (1)

where $y_f$ is $M \times 1$-dimensional observation data at the $f$-th frequency point. Note that $f = 1, \ldots, F$ is the
index in the frequency domain. The \( M \times K \)-dimensional matrix \( A_f = [a_f(\theta_1),...,a_f(\theta_K)] \) represents the array manifolds, where \( a_f(\theta_k) = [a_f(1,\theta_k),...,a_f(M,\theta_k)]^T \) and \( a_f(m,\theta_k) = e^{-j2\pi f(m-1)\cos \theta_k/c} \). The constant \( c \) is the sound propagation velocity. The \( K \times 1 \)-dimensional vector \( z_f = [z_f(1),...,z_f(K)]^T \) is composed of \( K \) sources, where \( z_f(k) \) is the data of the \( k \)-th source at the \( f \)-th frequency point. The vector \( n_f \) is the \( M \times 1 \)-dimensional measurement Gaussian noise.

In practical applications, the frequency characteristics of these \( K \) incident wideband sources may be different. The space-frequency map give in Fig. 1 shows a possible scenario of the position and frequency characteristics of sources in the spatial-frequency domain. The horizontal axis represents the selected frequency domain and the vertical axis represents the whole spatial domain. The frequency bands of sources may completely overlap, partially overlap, or non-overlap. In such a scenario, the DOA estimation algorithm that combines the whole frequency domain data is no longer applicable. Nevertheless, heterogeneous sources with different frequency characteristics still exhibit spatial sparsity, so we can implement DOA estimation through sparse representation. We divide the range of DOA into \( \Theta^{(\theta)} = [\theta_1,...,\theta_L] \), and assume that each source is located on the divided grid. Note that \( l = 1,...,L \) is the index in the spatial domain. Observation data \( y_f \) can then be expressed as a sparse model

\[
y_f = \Phi_f x_f + n_f, \quad f = 1,...,F,
\]

where the sparse coefficient vector \( x_f \in \mathbb{C}^{L \times 1} \) is \( K \)-sparse and there are \( K \) non-zero items that correspond to \( K \) values in \( z_f \). The matrix \( X = [x_1,...,x_F] \) is the sparse coefficient matrix and corresponds to the space-frequency map shown in Fig. 1, and will henceforth be called the space-frequency matrix. The \( M \times L \)-dimensional matrix \( \Phi_f = [a_f(\theta_1),...,a_f(\theta_L)] \) is the overcomplete dictionary at the \( f \)-th frequency point.

The DOA \( \{\theta_k\}_{k=1}^K \) and frequency characteristics of each source can be obtained by searching in the space-frequency matrix \( X \) for the non-zero rows and the non-zero elements within these rows. We find and record the index sets of the non-zero rows of \( X \) as

\[
\kappa = \{\tilde{l} \mid \|X_{\tilde{l}}\|_1 \neq 0\} = \{\tilde{l}_1,...,\tilde{l}_K\},
\]

where \( X_{\tilde{l}_k} \) is the \( \tilde{l}_k \)-th row of \( X \), and \( \| \cdot \|_1 \) denotes the \( \ell_1 \)-norm. Each non-zero row index has a one-to-one correspondence with the DOA \( \{\theta_k\}_{k=1}^K \) of the source such that

\[
\hat{\theta}_k = \Theta^{(\theta)}_{\hat{l}_k}, \quad \hat{l}_k \in \kappa, \quad k = 1,...,K,
\]

where \( \Theta^{(\theta)}_{\hat{l}_k} \) is the \( \hat{l}_k \)-th element in \( \Theta^{(\theta)} \). Similarly, the index sets of non-zero elements in each non-zero row in \( X \) is

\[
j_f(\hat{l}_k) = \{j(\hat{l}_k) \mid X_{\hat{l}_k,j(\hat{l}_k)} \neq 0\} = \{\hat{j}_1(\hat{l}_k),...,\hat{j}_{L}(\hat{l}_k)\}, \quad \hat{l}_k \in \kappa,
\]

where \( X_{\hat{l}_k,j(\hat{l}_k)} \) represents the \( (\hat{l}_k,\hat{j}(\hat{l}_k)) \)-th element of \( X \). Each non-zero element index has a one-to-one correspondence with the frequency characteristics of each source. The frequency characteristic of the \( k \)-th source obtained from the space-frequency matrix \( X \) is

\[
F_{\hat{j}_f(\hat{l}_k)} \in j_f(\hat{l}_k), \quad \hat{l}_k \in \kappa.
\]

Based on the above analysis, we can use a sparse representation method to solve \( X \) according to the spatial sparsity of sources. The DOA, number of sources and frequency characteristics of all sources are solved using (3)-(6). In addition, the value of each non-zero element in the \( k \)-th non-zero row corresponds to the signal amplitude of the \( k \)-th source at each frequency point. Note that only \( X \) is solved, based on the spatial sparsity. The frequency sparsity is not considered, because the frequency of the incident wideband source may cover the entire selected frequency band and is not sparse.

### III. ASF-SBL Algorithm for Space-Frequency Joint Processing

To address the heterogeneous wideband sources DOA estimation problem, we design an adaptive space-frequency structure prior model and propose a new space-frequency joint processing algorithm under SBL DOA framework (namely ASF-SBL algorithm). The new algorithm is designed for on-grid and off-grid DOA estimation. The following three parts are discussed in detail: (i) the adaptive space-frequency correlation structure prior model, (ii) the solution of the space-frequency joint processing algorithm, and (iii) the extension of the on-grid problem to off-grid problem.

#### A. Adaptive Space-Frequency Structure Prior Model

For the heterogeneous wideband sources DOA estimation, we design a space-frequency structure prior model to support the ASF-SBL algorithm. The introduction of spatial and frequency domains correlation has the following advantages: (i) improve the angular resolution of block sparse when sources are closely spaced, (ii) suppress spatial aliasing by selectively performing joint sparsity of data at each frequency point. In fact, not all coefficients are affected by their spatial-frequency domain neighbors. A constant space-frequency structural pattern prior will cause structural mismatch, resulting in erroneous DOA estimation results. To avoid this problem, an appropriate space-frequency structure prior model can be adaptively selected according to the structural pattern of the sparse signal itself.

1) **Space-Frequency Structural Patterns:** To effectively display the space-frequency structural pattern of each coefficient
in the sparse signal, we introduce the support matrix $S$ of the space-frequency matrix $X$. Each element in $S$ takes the value of 0 or 1. When the $(l, f)$-th coefficient in $X$ is non-zero, the value of the $(l, f)$-th element in $S$ is 1, otherwise it is 0. Note that $S_j$ and $S_{i,f}$ represent the $f$-th column and $(l, f)$-th element of $S$, respectively.

We realize the division of the space-frequency structural pattern according to the values of the current support element and its immediate neighbors in the spatial and frequency domains. Taking $S_{i,f}$ and its immediate neighbors $\{S_{i,f-1}, S_{i,f+1}, S_{i-1,f}, S_{i+1,f}\}$ as an example, all possible values of them are shown in Fig. 2, where the blue and white boxes indicate values of 1 and 0, respectively. According to these possible situations, Fig. 2 shows that we have divided the spatial (frequency) domain structural patterns into three categories:

- **Strong reception (S-recep):** the value of $S_{i,f}$ is the same as that of its spatial (frequency) domain neighbors.
- **Weak reception (W-recep):** the value of $S_{i,f}$ is the same as that of only one neighbor in the spatial (frequency) domain.
- **Strong rejection (S-rej):** the value of $S_{i,f}$ is different from that of its spatial (frequency) domain neighbors.

Since the structural patterns in both the spatial and frequency domains are divided into the above three categories, we can obtain nine space-frequency structural patterns as shown in Table I.

2) **The Prior of Support Matrix $S$:** To accurately estimate the support matrix $S$, [37], [39]–[41] reported that each support element should follow the Bernoulli distribution $S_{i,f} \sim \text{Bernoulli}(\gamma_{i,f})$ and its probability is $Pr(S_{i,f} = 1) = \gamma_{i,f}$ and $Pr(S_{i,f} = 0) = 1 - \gamma_{i,f}$. The assumption is made that the parameter $\gamma_{i,f}$ follows the Beta distribution, because the Beta distribution is a conjugate prior to the Bernoulli distribution. $S$ is controlled by $X$ to a certain extent, and its elements obey the following priors:

$$S_{i,f} \sim \text{Bernoulli}(\gamma_{i,f}),$$
$$\gamma_{i,f} \sim \text{Beta}(e + \|X_{i,f}\|_0, \|\hat{X}_{i,f}\|_0, h)$$

(7)

where $X_{i,f}$ is the $(l, f)$-th element of $X$. $\hat{X}_{i,f}$ represents the element set $\hat{X}_{i,f} = [X_{i-1,f}, X_{i,f}, X_{i+1,f}]$ consisting of $X_{i,f}$ and its spatial domain immediate neighbors. $\| \cdot \|_0$ represents the zero norm. The parameters $e$ and $h$ in the Beta distribution given by (7) control the probability $\gamma_{i,f}$ by interacting with different $\|X_{i,f}\|_0$ and $\|\hat{X}_{i,f}\|_0$. According to this prior, as shown in the dashed box in the spatial (frequency) domain structural pattern division in Fig. 2, we can divide the probability of support selection into the following four cases:

- When $\|X_{i,f}\|_0 = 1$ and $\|\hat{X}_{i,f}\|_0 = 0$, the probability $Pr(S_{i,f} = 1) = \gamma_{i,f}$ is strong.
- When $\|X_{i,f}\|_0 = 1$ and $\|\hat{X}_{i,f}\|_0 = 1$, there is an upper-middle probability $Pr(S_{i,f} = 1) = \gamma_{i,f}$.
- When $\|X_{i,f}\|_0 = 0$ and $\|\hat{X}_{i,f}\|_0 = 1$, there is a lower-middle probability $Pr(S_{i,f} = 1) = \gamma_{i,f}$.
- When $\|X_{i,f}\|_0 = 0$, regardless of whether the neighborhood is supported or not, a small probability $Pr(S_{i,f} = 1) = \gamma_{i,f}$ based on the previous updated $X_{i,f}$.

According to the above four possible probabilities, we set the parameters in the Beta distribution as $e = 1$ and $h = 4$. Subsequent experiments have verified that such values are feasible.

3) **Design of an Adaptive Space-Frequency Structure Prior Model:** The coefficients in $X$ are selectively affected by the space-frequency structural patterns of $S$. The ASP-SBL algorithm, therefore, includes a new sparse signal prior model, and the prior of the $(l, f)$-th coefficient $X_{i,f}$ of $X$ is given by

$$X_{i,f} \sim \text{CN}(0, \{V_{i,f} + \nu_{i,f}(V_{i-1,f} + V_{i+1,f}) + \nu_{i,f}(V_{i,f-1} + V_{i,f+1})\}^{-1})$$

(8)

The prior distribution of $X$ is

$$P(X|V, \nu, \nu) = \prod_{f=1}^{F} P(x_{j,f}|v_{f-1}, v_{f}, v_{f+1}, v_{f,j})$$

$$= \prod_{f=1}^{F} \prod_{t=1}^{L} \text{CN}(0, (\Lambda_{x_{j,t}, f}, v)$$

(9)

where

$$\Lambda_{x_{j,t}, f} = (V_{i,f} + \nu_{i,f}(V_{i-1,f} + V_{i+1,f}) + \nu_{i,f}(V_{i,f-1} + V_{i,f+1})^{-1}, \nu$$

and $\nu$ represent the correlation parameter matrix in the spatial and frequency domains, respectively. Each element in $\nu$ and $\nu$ satisfies $0 < \nu_{i,f}, \nu_{i,f} < 1$, where a large value indicates a strong correlation, and a small value indicates a weak correlation. $\nu_{i,f}$ and $\nu_{i,f}$ represent the $(l, f)$-th element of $\nu$ and $\nu$, respectively. Matrix $V$ represents the hyperparameter set corresponding to $X$, and $V_{i,f}$ and $V_{f}$ represent the $(l, f)$-th element and $f$-th column of $V$,
respectively. We impose the following gamma prior on \( V \):

\[
P(V) = \prod_{f=1}^{F} \prod_{l=1}^{L} \text{Gamma}(V_{lf}|c, d),
\]

where \( c \) and \( d \) should be assigned very small values to achieve non-information prior as commonly used in conventional SBL [25], [26]. By integrating hyperparameters, the sparse coefficient prior assumed to be “true” can be inferred as \( P(X_{lf}|V_{lf}) = \int P(X_{lf}|V_{lf})P(V_{lf})dV_{lf} \). The conditional density \( P(X_{lf}|V_{lf}) \) given in conventional SBL is \( X_{lf} \sim \text{CN}(0, V_{lf}^{-1}) \), then \( P(X_{lf}|V_{lf}) \propto (\frac{1}{2}X_{lf}^2 + d)^{-(\frac{d}{2}+c)} \). With \( c \) and \( d \) be very small values, such as \( c = d = 10^{-4} \), the prior reaches a strong peak near \( X_{lf} = 0 \), and there is a heavy tail. The small values of \( c \) and \( d \) influence sparsity and satisfy the purpose of sparse prior of the space-frequency matrix \( X \).

4) Adaptive Selection of Space-Frequency Structure Prior:
The flexible change of the adaptive space-frequency structure prior is realized by different elements in the correlation parameter matrices \( V \) and \( \nu \). \( \nu_{lf} \) and \( \nu_{lf} \) in \( V \) and \( \nu \) are not constant values, but are determined based on the space-frequency correlation pattern of \( S_{lf} \) and its immediate neighbors \( \{S_{l-1,f}, S_{l+1,f}, S_{l,f-1}, S_{l,f+1}\} \).

To determine the structural patterns in the spatial and frequency domains, we introduce \( P_{lf}^{(v)} \) and \( P_{lf}^{(v)} \) according to the structural pattern given in Fig. 2, where:

\[
P_{lf}^{(v)} = \sum_{s_{l,f}} + \sum_{s_{l,f}} + \sum_{s_{l,f}} + \sum_{s_{l,f}}\]

When \( P_{lf}^{(v)} = 2 \) and \( P_{lf}^{(v)} = 2 \), the structural pattern is determined to be strong reception; when \( P_{lf}^{(v)} = 1 \) and \( P_{lf}^{(v)} = 1 \), the structural pattern is weak reception; and when \( P_{lf}^{(v)} = 0 \) and \( P_{lf}^{(v)} = 0 \), the structural pattern is strong rejection. For strong reception, \( \nu_{lf} = \nu \) and \( \nu_{lf} = \nu \) are chosen; for weak reception, \( \nu_{lf} = \nu/5 \) and \( \nu_{lf} = \nu/5 \) are chosen; for strong rejection, \( \nu_{lf} = 0 \) and \( \nu_{lf} = 0 \) are chosen. Since \( \nu_{lf} \) and \( \nu_{lf} \) are selected according to the structural pattern of \( S \), their priors are given by

\[
P(V|S) = \prod_{f=1}^{F} \prod_{l=1}^{L} P(\nu_{lf}|S_{lf}) = \prod_{f=1}^{F} \prod_{l=1}^{L} N(\nu_{lf}, \delta_{v}),
\]

where \( \nu_{lf} \) is the received data in the frequency domain. The Gamma distribution \( P(\alpha_0) = \text{Gamma}(a, b) \) is the prior of \( \alpha_0 \). The parameters \( a \) and \( b \) are usually chosen to be small values to provide non-information prior. In the subsequent simulations, we will take \( a = b = 10^{-4} \) as in [25].

Let \( \Theta = \{S, X, \gamma, V, \alpha_0, \nu, \nu\} \) be the hidden variable set to be estimated. Fig. 3 is the graphical association model of the ASF-SBL algorithm, which shows the relationship between the variables. According to the figure, the joint probability density function (PDF) is

\[
P(Y, S, X, \gamma, V, \alpha_0, \nu, \nu) = P(Y|X, \alpha_0)P(X|V, \nu, \nu) \\
\times P(V)P(S|\gamma)P(\alpha_0)P(\gamma|X)P(\nu|S)P(\nu|S).
\]

In summary, the adaptive selection of the space-frequency structure prior is achieved through the following process. We first solve \( P_{lf}^{(v)} \) and \( P_{lf}^{(v)} \), using the support element \( S_{lf} \) and its immediate neighbors \( \{S_{l-1,f}, S_{l+1,f}, S_{l,f-1}, S_{l,f+1}\} \). Then, according to \( P_{lf}^{(v)} \) and \( P_{lf}^{(v)} \), correlation parameters \( \nu_{lf} \) and \( \nu_{lf} \) are obtained from (11). Finally, the estimated \( \nu_{lf} \) and \( \nu_{lf} \) are substituted into (8) to obtain the prior of the current sparse coefficient \( X_{lf} \). The prior model shown in (8) can be changed according to different space-frequency structural patterns, so that it can be applied to each coefficient in the space-frequency matrix \( X \).

B. Solution of the Space-Frequency Joint Processing Algorithm

The previous section introduced the adaptive space-frequency structure prior model of the sparse coefficient designed in the ASF-SBL algorithm. We now incorporate the model in the SBL framework to solve the space-frequency matrix \( X \) to achieve DOA estimation.

Suppose that the additive noise \( n_f \) in the array receiving model follows a complex Gaussian distribution \( \text{CN}(0, \alpha_0^{-1}) \). The likelihood function of (2) is then given by

\[
P(Y|X, \alpha_0) = \prod_{f=1}^{F} \text{CN}(y_f|x_f, \alpha_0) = \prod_{f=1}^{F} \text{CN}(\Phi_f x_f, \alpha_0^{-1} I),
\]

where \( Y = [y_1, \ldots, y_F] \) is the received data in the frequency domain. The Gamma distribution \( P(\alpha_0) = \text{Gamma}(a, b) \) is the prior of \( \alpha_0 \). The parameters \( a \) and \( b \) are usually chosen to be small values to provide non-information prior. In the subsequent simulations, we will take \( a = b = 10^{-4} \) as in [25].

Let \( \Theta = \{S, X, \gamma, V, \alpha_0, \nu, \nu\} \) be the hidden variable set to be estimated. Fig. 3 is the graphical association model of the ASF-SBL algorithm, which shows the relationship between the variables. According to the figure, the joint probability density function (PDF) is

\[
P(Y, S, X, \gamma, V, \alpha_0, \nu, \nu) = P(Y|X, \alpha_0)P(X|V, \nu, \nu) \\
\times P(V)P(S|\gamma)P(\alpha_0)P(\gamma|X)P(\nu|S)P(\nu|S).
\]
For the solution of $\Theta = \{\mathbf{S}, \mathbf{X}, \gamma, \mathbf{V}, \alpha_0, \nu, \nu\}$, we refer to the variational Bayesian inference (VBI) given in [42], [43]. In this inference method, the Kullback-Leibler (KL) divergence $D_{KL}(q(\Theta)||p(\Theta|\mathbf{Y}))$, which describes the difference between the approximate posterior $q(\Theta) = q(\mathbf{S}|q(\mathbf{X})q(\gamma)q(\mathbf{V})q(\alpha_0)q(\nu)q(\nu))$ of $\Theta$ and the posterior PDF $p(\Theta|\mathbf{Y})$, is used to calculate $q(\Theta)$ instead of directly estimating $p(\Theta|\mathbf{Y})$. Refs. [42], [43] have deduced that the optimal distribution of $q(\Theta_i)$ is

$$ln q'_{\ast}(\Theta_i) = \langle ln P(\mathbf{Y}, \Theta) \rangle_{q(\Theta_i)\langle \Theta_i \rangle} + const.$$  

(14)

where $\langle \cdot \rangle_{q(\Theta_i)\langle \Theta_i \rangle}$ indicates the expectation with respect to $q(\Theta\setminus \Theta_i)$ and $\Theta\setminus \Theta_i$ represents the hidden variables set $\Theta$ without $\Theta_i$.

We introduce the new prior models described by (7)–(12) designed for theASF-SBL algorithm, and derive $\Theta = \{\mathbf{S}, \mathbf{X}, \gamma, \mathbf{V}, \alpha_0, \nu, \nu\}$ based on the optimal distribution given by (14). The derivation process of the algorithm is as follows, where the updated result of each hidden variable is recorded as $\langle \cdot \rangle$ in each update process.

1) Update of $\mathbf{X}$: According to (14),

$$ln q(\mathbf{X}) \propto \langle ln P(\mathbf{Y}, \mathbf{S}, \mathbf{X}, \gamma, \mathbf{V}, \alpha_0, \nu, \nu) \rangle_{q(\Theta)\langle \mathbf{X} \rangle}$$

$\propto \langle ln P(\mathbf{Y}|\mathbf{X}, \alpha_0)P(\mathbf{X}|\mathbf{V}, \nu)P(\gamma|\mathbf{V})q(\nu)q(\nu|\alpha_0) \rangle.$

Substituting (7), (9), and (12) into the above equation gives

$$ln q(\mathbf{x}_f) \propto -\alpha_0 \|\mathbf{y}_f - \Phi_f \mathbf{x}_f\|^2 - (\mathbf{x}_f)\mathbf{H}_f^\dagger \mathbf{A}_{x_f}^{-1} \mathbf{x}_f$$

$$+ \sum_{l=1}^{L} \left( (\epsilon + X_{l,f} + X_{l,f}) \right) \ln \gamma_{l,f}$$

$$+ (\lambda - X_{l,f}) \|\mathbf{x}_{l,f}\|_0 \ln (1 - \lambda_{l,f}).$$

The last term of the above equation selects the previously updated $\{X_{l,f}, X_{l-1,f}, X_{l+1,f}\}$, which can be regarded as a constant, so only the first two terms are considered. The approximate posterior of $\mathbf{x}_f$ is then a complex Gaussian distribution given by $q(\mathbf{x}_f) \propto \exp(-\langle \mathbf{x}_f - \mu_{x_f} \rangle\mathbf{H}_f^\dagger \mathbf{A}_{x_f}^{-1} \mathbf{x}_f)$, where the variance $\Sigma_{x_f}$ and mean $\mu_{x_f}$ are

$$\Sigma_{x_f} = (\alpha_0 \Phi_f^\dagger \Phi_f + \mathbf{A}_{x_f}^{-1})^{-1},$$

$$\mu_{x_f} = (\Phi_f^\dagger \Phi_f + \alpha_0^{-1} \mathbf{A}_{x_f}^{-1})^{-1} \Phi_f^\dagger \mathbf{y}_f.$$  

(15)

Finally, the update rule of $\mathbf{x}_f$ is given by

$$\hat{x}_f = \mu_{x_f} = \alpha_0 \Sigma_{x_f} \Phi_f^\dagger \mathbf{y}_f, f = 1, \ldots, F.$$  

(16)

2) Update of $\alpha_0$: According to (14),

$$ln q(\alpha_0) \propto \langle ln P(\mathbf{Y}, \mathbf{S}, \mathbf{X}, \gamma, \mathbf{V}, \alpha_0, \nu, \nu) \rangle_{q(\Theta)\langle \alpha_0 \rangle}$$

$\propto \langle ln P(\mathbf{Y}|\mathbf{X}, \alpha_0)P(\mathbf{X}|\mathbf{V}, \nu)P(\gamma|\mathbf{V})q(\nu)q(\nu|\alpha_0) \rangle.$

Substituting (12) and $p(\alpha_0) \sim \text{Gamma}(a, b)$ into the above equation, and using the relation $\langle \mathbf{x}_f^\dagger \mathbf{H}_f^\dagger q(\mathbf{x}_f) \rangle = \mu_{x_f} \mu_{x_f}^\dagger + \Sigma_{x_f}$, gives:

$$ln q(\alpha_0) \propto - \left( b + \sum_{f=1}^{F} \langle \|\mathbf{y}_f - \Phi_f \mathbf{x}_f\|^2/q(\mathbf{x}_f) \rangle / 2 \right) \alpha_0$$

$$+ (a - 1 + FM/2) \ln \alpha_0$$

$$\propto \left( b + \sum_{f=1}^{F} \langle \|\mathbf{y}_f - \Phi_f \mu_{x_f}\|^2 / 2 \right) + \text{Tr}(\Phi_f^\dagger \Phi_f \Sigma_{x_f}) \right) \alpha_0$$

$$+ (a - 1 + FM/2) \ln \alpha_0.$$  

(17)

The approximate posterior distribution of $\alpha_0$, therefore, follows a Gamma distribution and the updated $\alpha_0$ is given by

$$\hat{\alpha}_0 = \frac{2(a - 1) + FM}{2b + \sum_{f=1}^{F} \langle \|\mathbf{y}_f - \Phi_f \mathbf{x}_f\|^2 / 2 \right) + \text{Tr}(\Phi_f^\dagger \Phi_f \Sigma_{x_f}) \rangle}.$$  

(18)

3) Update of $\mathbf{V}$: For $q(\mathbf{V})$, we have

$$ln q(\mathbf{V}) \propto \langle ln P(\mathbf{Y}, \mathbf{S}, \mathbf{X}, \gamma, \mathbf{V}, \alpha_0, \nu, \nu) \rangle_{q(\Theta)\langle \mathbf{V} \rangle}$$

$\propto \langle ln P(\mathbf{X}|\mathbf{V}, \nu)P(\mathbf{V})q(\nu)q(\nu|\mathbf{X}) \rangle.$

Only items related to $V_{l,f}$ are retained such that

$$ln q(V_{l,f}) \propto (c - 1) \ln V_{l,f} - dV_{l,f} + \left( \langle \Lambda_x^f \rangle_{l,l} \right)^{-1} \left( \langle \Lambda_x^f \rangle_{l,l} \right) - \left( \langle \Lambda_x^f \rangle_{l,l} \right)^{-1} V_{l,f} \left( \langle \Lambda_x^f \rangle_{l,l} \right)^{-1} V_{l,f}$$

where $h_{i,l} = (\mu_{x_f})^2 + (\Sigma_{x_f})_{l,l}$.

Unfortunately, the optimal solution of $V_{l,f}$ cannot be obtained because $V_{l,f}$ is entangled with the unknowns $\{V_{l-1,f}, V_{l+1,f}, V_{l-1,f}, V_{l+1,f}\}$. Since all hyperparameters in $\nu$ are non-negative, we have $\frac{1}{V_{l,f}} > (\Lambda_x^f)_{l,l} > 0$, $\frac{1}{V_{l-1,f}} > (\Lambda_x^f)_{l,l} > 0$, and $\frac{1}{V_{l+1,f}} > (\Lambda_x^f)_{l,l} > 0$. After selecting the upper bounds, the sub-optimal solution of $V_{l,f}$ is given by

$$\hat{V}_{l,f} = \frac{2(a - 1) + 5}{2d + \text{Tr}(\Sigma)}.$$  

(20)

where $\Sigma = h_{i,l}^f + \hat{v}_{l-1,f}h_{i-1,l}^f + \hat{v}_{l+1,f}h_{i+1,l}^f$.
otherwise extremely large values will affect the calculation of hyperparameters.

4) Update of S: According to (14),

\[
\ln q(S) \propto \langle \ln P(Y, S, X, \gamma, V, \alpha_0, \nu, \upsilon) \rangle_{q(\Theta)}q(S) = \langle \ln P(S|\gamma)P(\nu|S)P(\upsilon|S) \rangle_{q(\gamma)}q(\nu)q(\upsilon).
\]

Let’s consider only \( S_{t,f} \). Since \( S \) in \( P(\nu|S) \) and \( P(\upsilon|S) \) is the result of the previous update and can be recorded as a constant, substituting (7) and (11) into the equation above gives

\[
\ln q(S_{t,f}) \propto \langle \ln P(S_{t,f}|\gamma) \rangle_{q(\gamma,t)},
\]

\[
\propto (S_{t,f} \ln \gamma_{t,f} + (1 - S_{t,f}) \ln(1 - \gamma_{t,f})) q(\gamma,t).
\]

The probabilities of \( S_{t,f} = 1 \) and \( S_{t,f} = 0 \) are given by

\[
P(S_{t,f} = 1) \propto \exp(\ln(\hat{\gamma}_{t,f})),
\]

\[
P(S_{t,f} = 0) \propto \exp(\ln(1 - \hat{\gamma}_{t,f})).
\]

Finally, the update rule of \( \{S_{t,f}\}_{t=1:F, f=1:L} \) is

\[
\hat{S}_{t,f} = \frac{P(S_{t,f} = 1)}{P(S_{t,f} = 0) + P(S_{t,f} = 1)}.
\]

5) Update of \( \gamma \): According to (14), the optimal distribution of \( q(\gamma) \) is

\[
\ln q(\gamma) \propto \langle \ln P(Y, X, S, \gamma, V, \alpha_0, \nu, \upsilon) \rangle_{q(\Theta)}q(\gamma) = \langle \ln P(S|\gamma)P(\gamma|X)q(X)q(\gamma) \rangle_{q(X)}q(\gamma).
\]

For \( \gamma_{t,f} \),

\[
\ln q(\gamma_{t,f}) \propto \langle \ln P(S_{t,f}|\gamma)P(\gamma_{t,f}|\hat{X}_{t,f})q(\hat{X}_{t,f})q(S_{t,f}) \rangle_{q(S_{t,f})}
\]

It can be seen that \( \gamma_{t,f} \) obeys the Beta distribution \( \text{Beta}(\hat{\gamma}, \hat{\nu}) \),

\[
\hat{\gamma}_{t,f} = \hat{S}_{t,f} + e + \hat{X}_{t,f}||\hat{X}_{t,f}||_0, \\
\hat{\nu}_{t,f} = 1 - \hat{S}_{t,f} + h + \hat{X}_{t,f}||\hat{X}_{t,f}||_0.
\]

According to (21), the intermediate variables \( \langle \ln(\gamma_{t,f}) \rangle \) and \( \langle \ln(1 - \gamma_{t,f}) \rangle \) must be solved to calculate the target matrix \( S \). Taking the derivative of (23) with respect to \( \hat{\gamma}_{t,f} \) and \( \hat{\nu}_{t,f} \), and setting each first derivative equal to zero gives

\[
\frac{\partial \ln q(\gamma_{t,f})}{\partial \hat{\gamma}_{t,f}} = \psi(\hat{\gamma}_{t,f}) - \psi(\hat{\nu}_{t,f}) + \ln(\gamma_{t,f}) = 0,
\]

\[
\frac{\partial \ln q(\gamma_{t,f})}{\partial \hat{\nu}_{t,f}} = \psi(\hat{\gamma}_{t,f}) - \psi(\hat{\nu}_{t,f}) + \ln(1 - \gamma_{t,f}) = 0.
\]

In the above equation, \( \psi(\cdot) = \frac{d}{d t} \ln(\Gamma(\cdot)) \) is the digamma function, \( \langle \ln(\gamma_{t,f}) \rangle \) and \( \langle \ln(1 - \gamma_{t,f}) \rangle \) can be expressed as

\[
\langle \ln(\gamma_{t,f}) \rangle = \psi(\hat{\gamma}_{t,f}) - \psi(\hat{\gamma}_{t,f} + \hat{\nu}_{t,f}), \\
\langle \ln(1 - \gamma_{t,f}) \rangle = \psi(\hat{\nu}_{t,f}) - \psi(\hat{\gamma}_{t,f} + \hat{\nu}_{t,f}).
\]

\( \hat{\gamma}_{t,f} \) and \( \hat{\nu}_{t,f} \) are positive integers and it is well known that \( \psi(x + 1) = \psi(x) + \frac{1}{x} \) and \( \psi(1) \) is Euler’s constant –0.5772. Equation (25) can, therefore, be calculated to obtain the four probability possibilities in the dashed box in Fig. 2. Finally, \( S_{t,f} \) can be solved by substituting (25) into (21) and (22). We set the threshold \( \text{thres} = 0.4 \) by taking into account the (i) previous updated \( X \) as the constraint of the support calculation, and (ii) four probability cases of the Beta distribution. If \( \hat{S}_{t,f} \geq \text{thres} \), \( \hat{S}_{t,f} \) is updated to 1, otherwise \( \hat{S}_{t,f} \) is updated to 0.

6) Update of \( \nu, \upsilon \): Using (9), (11) and (14), combined with the fact that \( \nu \) and \( \upsilon \) in \( P(X|V, \nu, \upsilon) \) are the previously updated values, allows the optimal distribution of \( q(\nu) \) and \( q(\upsilon) \) to be obtained as

\[
\ln q(\nu) \propto \langle \ln P(X|V, \nu, \upsilon)P(\nu|S)q(X)q(\upsilon)q(S) \rangle_{q(S)}q(\nu),
\]

\[
\ln q(\upsilon) \propto \langle \ln P(X|V, \nu, \upsilon)P(\upsilon|S)q(X)q(\nu)q(S) \rangle_{q(S)}q(\upsilon).
\]

Only items related to \( \nu_{t,f} \) are retained, so

\[
\ln q(\nu_{t,f}) \propto \frac{1}{2} \left( \ln(\Lambda_{x,t}^{\nu} + (\Lambda_{x,t}^{\nu} - h_{t,f} - 1)^{\nu_{t,f} - \mu_{t,f}^{\nu}}) - \frac{(\nu_{t,f} - \mu_{t,f}^{\nu})^2}{2\nu} \right).
\]

Now, letting \( \partial \ln q(\nu_{t,f})/\partial \nu_{t,f} = 0 \) gives \( \frac{d}{d t}(\nu_{t-1,f} + V_{t+1,f})(\Lambda_{x,t} - h_{t,f}) + (\nu_{t,f} - \mu_{t,f}^{\nu}) = 0. \) \( \delta \nu \) is a small value that tends to zero, which can be ignored. The update rule of \( \nu_{t,f} \) is, therefore, given by

\[
\nu_{t,f} = \mu_{t,f}^{\nu}.
\]

Similarly, the update rule of \( \upsilon_{t,f} \) is

\[
\upsilon_{t,f} = \mu_{t,f}^{\upsilon}.
\]

Algorithm 1 summarizes the detailed steps of the ASF-SBL algorithm for the wideband DOA estimation problem. Ref. [43] has verified the convergence of VBI, so the ASF-SBL algorithm, which also uses VBI to solve hidden variables, can converge to a local optimum as the iterative process is executed. For this reason, the error \( \varepsilon = ||X_{x,t}^{(t)} - X_{x,t}^{(t-1)}||_F/||X_{x,t}^{(t-1)}||_F \) of two successive estimations of \( X_{x,t} = [\mu_{x,1}, \ldots, \mu_{x,L}] \) will be used to assess convergence in this work. The algorithm is deemed to have converged when (i) \( \varepsilon \) is less than a given threshold \( \varepsilon_{\text{err}} \) or (ii) the number of iterations exceeds the predefined maximum number of iterations \( T_{\text{max}} \). After convergence, the final result of \( X_{x,t}^{(t)} \) is taken as the optimal solution of the ASF-SBL algorithm, which is the estimated space-frequency matrix \( X \). Finally, the DOA and frequency characteristics of each source can be obtained from \( X \) using (3)-(6).

C. Extension of the On-Grid Problem to the Off-Grid Problem

The ASF-SBL algorithm is an on-grid DOA estimation algorithm, i.e., it is implemented based on the assumption that the source is located on a grid divided in the spatial domain. However, the off-grid problem is usually encountered in DOA estimation, that is, the source often does not lie exactly on the grid. This subsection outlines the off-grid model and provides details of the process for solving the off-grid problem within the framework of the ASF-SBL algorithm.

1) Off-Grid Mathematical Model: Suppose the true DOA of the \( k \)-th source is \( \theta_k \notin \{\theta_k \}_{l=1:L} \) and \( \theta_{l_k} \) is the grid closest to \( \theta_k \), where \( l_k \in [1, L] \). Refs. [44], [45] derived the steering vector of the array receiving mathematical model, which can be expressed
Algorithm 1: The Framework of the ASF-SBL Algorithm.

Input: $Y$, $\{\Phi_f\}_{f=1:F}$, $\{\nu, \alpha, b, c, d, e, h\}$, $\delta_a, \delta_b, T_{\text{max}}$, $\varepsilon_{\text{err}}$. Initialize: $\alpha_0^{(0)}, S(0)^{(0)}, \gamma_0^{(0)}, V(0), \{\Delta_x^{(0)}\}_{f=1:F}$.

Output: space-frequency map $X$, DOA, and frequency characteristics of each source.

1: while $\varepsilon_{\text{err}} > \varepsilon$ and $t < T_{\text{max}}$ do
2: \hspace{1cm} $t = t + 1$;
3: \hspace{1cm} for $f = 1, \ldots, F$ do
4: \hspace{1.5cm} Calculate $\mu_x^{(t)}$ and $\Sigma_x^{(t)}$ by (15);
5: \hspace{1.5cm} Calculate $\alpha_0^{(t)}$ by (18);
6: \hspace{1.5cm} Calculate $\{\nu_l^{(t)}\}_{l=1:L}$ by (20);
7: \hspace{1.5cm} Calculate $\hat{\varepsilon}$, and $\hat{h}_r$ by (24), and obtain $\ln(\gamma_l f)$ and $\ln(1 - \gamma_l f)$, $l = 1, \ldots, L$ by (25);
8: \hspace{1.5cm} Calculate $\{\gamma_0^{(t)}\}_{l=L}^{1}$ by (21) and (22);
9: \hspace{1.5cm} Calculate $\{v_l^{(t)}, v_{l,f}^{(t)}\}_{l=1:L, f}$ by (26) and (27);
10: \hspace{0.5cm} end for
11: $X_{\mu}^{(t)} = [\mu_x^{(t)}], \ldots, \mu_x^{(t)}$,
12: \hspace{1.2cm} $\varepsilon = ||X_{\mu}^{(t)} - X_{\mu}^{(t-1)}||_F/||X_{\mu}^{(t-1)}||_F$.
13: end while
14: return $X = X_{\mu}^{(t)}$, and calculate DOA and frequency characteristics of each source by performing (3)-(6) on the estimated $X$.

\[
\Phi_f(\theta_k) \approx \Phi_f(\hat{\theta}_k) + \Phi_f(\hat{\theta}_k)(\theta_k - \hat{\theta}_k) \\
= \Phi_f(\hat{\theta}_k) + \Phi_f(\hat{\theta}_k)\delta_l,
\]

where the off-grid deviation is
\[
\delta_l = \left\{ \begin{array}{ll}
\theta_k - \hat{\theta}_k, & \text{if } l = k, \text{for any } k \in [1, \ldots, K]; \\
0, & \text{otherwise},
\end{array} \right.
\]
\[
\bar{\delta} = [\delta_1, \ldots, \delta_K]^T \quad \text{and} \quad \Delta = \text{diag}(\bar{\delta}) \quad \text{are recorded as the deviation vector and the deviation matrix, respectively.} \]

Based on the above analysis, we construct the array receiving space-frequency map DOA estimation by compensating $\Delta$. The likelihood function of (29) is given by
\[
P(Y|X, \alpha_0, \Delta) = \prod_{f=1}^{F} C N(\Psi_f X_f, \alpha_0^{-1} I).
\]

2) Solution of the Off-Grid Problem: We use VBI to update all parameters. In the off-grid problem, the parameters to be estimated are added $\Delta$ on the basis of the original hidden variables $\Theta = \{S, X, \gamma, V, \alpha_0, \nu, v\}$ in Section III.B. Then, the joint PDF is
\[
P(Y, S, X, \gamma, V, \alpha_0, \nu, v, \Delta) \\
= P(Y|X, \alpha_0, \Delta) P(X|\nu, v) \\
\times P(V|S) P(\gamma|X) P(\alpha_0) P(\nu|S) P(v|S) P(\Delta).
\]

Since the update rules of the part hidden variables $\{S, \gamma, V, \nu, v\}$ are not affected by the new variable $\Delta$, their updates remain the same as the update rules derived in Section III.B. For this reason, only the update solutions of the remaining variables $\{X, \alpha_0, \Delta\}$ are discussed below.

3) Update of $X$: According to (31),
\[
\ln q(x_f) \propto -\alpha_0 |y_f - \Psi_f x_f|^2 - (x_f)^H \Delta_{x_f} x_f.
\]

Finally, $x_f$ can be updated by
\[
\hat{x}_f = \mu_{x_f} = \alpha_0 \Sigma_{x_f} \Psi_f^H y_f, \quad f = 1, \ldots, F.
\]

4) Update of $\alpha_0$: For $q(\alpha_0)$, we can obtain
\[
\ln q(\alpha_0) \propto \ln P(Y|X, \alpha_0, \Delta) P(\alpha_0) q(\Delta) q(X) \\
\propto (a - 1 + FM/2) \ln \alpha_0 - \left( b + \sum_{f=1}^{F} |y_f - \Psi_f \mu_{x_f}|^2 + \text{Tr}(\Psi_f^H \Sigma_{x_f}) \right) \alpha_0.
\]

The approximate posterior distribution of $\alpha_0$ follows a Gamma distribution, and the update rule of $\alpha_0$ is given by
\[
\hat{\alpha}_0 = \frac{2(a - 1) + FM}{2b + \sum_{f=1}^{F} |y_f - \Psi_f \mu_{x_f}|^2 + \text{Tr}(\Psi_f^H \Sigma_{x_f})}.
\]

5) Update of $\Delta$: According to (31),
\[
\ln q(\Delta) \propto \ln P(Y|X, \alpha_0, \Delta) P(\Delta) q(\alpha_0) q(X) \\
\propto \ln P(\Delta) - \hat{\alpha}_0 \sum_{f=1}^{F} |y_f - \Psi_f \mu_{x_f}|^2 + \text{Tr}(\Psi_f^H \Sigma_{x_f})
\]

where $s_f = \text{diag}(S_f)$, $B_f = \Phi_f s_f$, and $\Psi_f = \Phi_f + B_f \Delta$.
Expanding the last two terms of the equation above gives
\[ ||y_f - \Psi_f \mu_{x_f}||^2 = const + \delta^T \left( (B^H_f B_f)^* \mu_{x_f} \mu_{x_f}^H \right) \delta - 2 \left( \text{diag}(\mu_{x_f}^*) B_f^H (y_f - \Phi_f \mu_{x_f}) \right)^T \delta, \]
\[ \text{Tr}(\Psi^H_f \Psi_f \Sigma_{x_f}) = 2 \left( \text{diag}(B^H_f \Phi_f \Sigma_{x_f}) \right)^T \delta + \delta^T \left( \Sigma_{x_f} \otimes (B^H_f B_f)^* \right) \delta + const, \]
where \((\cdot)^*\) denotes the complex conjugate of a matrix. Substituting the above results into (37) and setting the derivation of \(\ln q(\Delta)\) with respect to \(\delta\) equal to zero gives \(2 \Xi \delta - 2 \Omega = 0\), where
\[ \Xi = \sum_{f=1}^{F} \left( (B^H_f B_f)^* \otimes (\mu_{x_f} \mu_{x_f}^H + \Sigma_{x_f}) \right), \]
\[ \Omega = \sum_{f=1}^{F} \left( \text{diag}(\mu_{x_f}^*) B_f^H (y_f - \Phi_f \mu_{x_f}) - \text{diag}(B_f^H \Phi_f \Sigma_{x_f}) \right). \]
If \(\Xi\) is reversible, the update rule of \(\delta\) is
\[ \delta = \Xi^{-1} \Omega. \quad (38) \]
Otherwise, each element in \(\delta\) must be updated. \(\delta_l \sim \left[ -\frac{\theta_{\text{dx},i}}{2}, \frac{\theta_{\text{dx},i}}{2} \right] \) such that \(\delta_l\) can be updated according to
\[ \delta_l = \begin{cases} \frac{\Omega_l - (\Xi_l)^{(\text{row})}}{\Xi_l}, & \text{if } \delta_l \in \left[ -\frac{\theta_{\text{dx},i}}{2}, \frac{\theta_{\text{dx},i}}{2} \right], \\ -\frac{\theta_{\text{dx},i}}{2}, & \text{if } \delta_l < -\frac{\theta_{\text{dx},i}}{2}, \\ \frac{\theta_{\text{dx},i}}{2}, & \text{if } \delta_l > \frac{\theta_{\text{dx},i}}{2}, \end{cases} \quad (39) \]
where \((\cdot)^{(\text{row})}\) represents the vector composed of the elements that remain after removing the \(l\)-th element from the vector in the brackets, \((\cdot)^{(\text{row})}\) represents the \(l\)-th row of the matrix.

In summary, the off-grid DOA estimation is solved within the framework of Algorithm 1. In the algorithm, the update rules of the hidden variables \(X\) and \(\alpha\) are replaced by (34) and (36), respectively. The update rule for the unknown \(\delta\), detailed in (38) and (39), is added to the loop part of the algorithm. Finally, \(\delta\) is incorporated into the estimation result obtained by performing (3)-(4) on the estimated \(X\) to realize the off-grid DOA estimation.

IV. TWO-DIMENSIONAL DOA ESTIMATION USING THE ASF-SBL ALGORITHM

In this section, we extend the ASF-SBL algorithm to 2-D DOA estimation. Note that 2-D DOA can be uniquely determined based on the space angles of two intersecting ULAs [46], [47]. We design a cross array composed of two orthogonal ULAs as shown in Fig. 4 as a signal receiving device. The ASF-SBL algorithm can be used to calculate the space angles for each ULA, which can in turn be used to calculate the 2-D DOA.

The two sets of 1-D space angles obtained through distributed processing may not be arranged in the order corresponding to the \(K\) incident sources, it is necessary to match the two sets of space angles. For this reason, the following discussion is split into two parts: (i) 2-D DOA estimation for distributed processing and (ii) space angle matching.

A. 2-D DOA Estimation for Distributed Processing

To implement 2-D DOA estimation, we design the cross array shown in Fig. 4 and introduce space angles \(\{\alpha, \beta\}\), where \(\alpha\) and \(\beta\) are the angles between the incident direction of the \(k\)-th source and the positive \(x\)-axis direction and the positive \(y\)-axis direction, respectively.

For the solution of \(\{\alpha, \beta\} = 1:K\), we first divide the ranges of space angle into \(\Theta_\alpha = [\alpha_1, ..., \alpha_{L_\alpha}]\) and \(\Theta_\beta = [\beta_1, ..., \beta_{L_\beta}]\). A sparse model, similar to (2), is constructed for the two ULAs of the cross array. The two models are then processed using the ASF-SBL algorithm to obtain the space-frequency matrix \(X^x(\theta)\) in the \(x\)-axis ULA model and the space-frequency matrix \(X^y(\theta)\) in the \(y\)-axis ULA model. Finally, the angles corresponding to the non-zero row index sets \(\Theta_\alpha^\text{set}\) and \(\Theta_\beta^\text{set}\) of \(X^x(\theta)\) and \(X^y(\theta)\) obtained by (3)-(4) give the space angles \(\{\alpha_k, \beta_k\} = 1:K\).

The relationship between DOA \(\{\theta_\theta, \phi_k\}\) and space angle \(\{\alpha_k, \beta_k\}\) can be obtained from Fig. 4 as \(\cos \alpha_k = \cos \theta_k \cos \phi_k\) and \(\cos \beta_k = \sin \theta_k \cos \phi_k\). Based on the above relationship, the DOA of each source is given by
\[ \theta_k = \arctan \left( \frac{\cos \beta_k}{\cos \alpha_k} \right), k = 1, ..., K, \]
\[ \phi_k = \arccos \sqrt{\cos^2 \alpha_k + \cos^2 \beta_k}, \]
where the angular ranges of the estimated azimuth and elevation angle are \(\theta_k \in [0^\circ, 180^\circ]\) and \(\phi_k \in [0^\circ, 90^\circ]\), respectively. In practice, the estimated range of the azimuth angle is expected to be \(\theta_k \in [0^\circ, 360^\circ]\), so more information needs to be introduced.

Table II shows the sign information of \(\cos \alpha_k\) and \(\cos \beta_k\) in different angular quadrants. The sign of \(\cos \alpha_k\) is “+” when \(\theta_k \in [0^\circ, 180^\circ]\), and the sign of \(\cos \beta_k\) is “−” when \(\theta_k \in [181^\circ, 360^\circ]\). The azimuth angle in the complete range can, therefore, be distinguished according to the sign information of \(\cos \beta_k\). Finally,
the 2-D DOA estimation is given by

\[
\theta_k = \begin{cases} 
\arctan\left(\frac{\sin \beta_k}{\cos \alpha_k}\right), & \text{if } \cos \beta_k \geq 0, \\
\arctan\left(\frac{\cos \beta_k}{\cos \alpha_k}\right) + 180^\circ, & \text{if } \cos \beta_k < 0,
\end{cases}
\]

\[
\phi_k = \arccos \sqrt{\cos^2 \alpha_k + \cos^2 \beta_k}.
\] (40)

**B. Space Angle Matching**

2-D DOA estimation can be achieved by converting the estimated space angle through (40). Before the angle conversion, it is necessary to match the space angle results. This is mainly because the orders of the non-zero row index sets \(\kappa(x)\) and \(\kappa(y)\) of the matrices \(X(x)\) and \(X(y)\) may not correspond to the order of the sources in the spatial domain. This non-correspondence leads to errors in the DOA estimation.

As shown in Fig. 5, the frequency characteristics of sources present three cases: a) the frequency characteristics of each source are different, b) there are several sources with the same frequency characteristics, or c) the signal powers of sources with the same frequency characteristics are similar. The most direct method to match the space angles \(\{\alpha_k\}_{k=1:K}\) and \(\{\beta_k\}_{k=1:K}\) is to pair non-zero row indexes with the same frequency characteristics in \(X(x)\) and \(X(y)\). This method of space angle matching is only applicable to case a). If the frequency characteristics of several sources are the same, that is, in case b), the correct combination of \(\alpha_k\) and \(\beta_k\) cannot be obtained through direct observation of the frequency characteristics. The value of the non-zero element in each non-zero row of the space-frequency matrix corresponds to the signal amplitude of each source at each frequency point. Therefore, the space angles \(\{\alpha_k\}_{k=1:K}\) and \(\{\beta_k\}_{k=1:K}\) corresponding to the similar signal amplitudes in \(X(x)\) and \(X(y)\) can be matched and grouped in case c). For case c), the number of sources in the selected frequency band is known from the space-frequency map. The MUSIC algorithm can, therefore, be used to perform a 2-D search to obtain the paired space angles. The alternative method used for case c) achieves the purpose of space angle matching. However, it is worth noting that this situation is not common.

**V. DISCUSSION**

**A. The Advantages of the Adaptive Prior Model**

In the ASF-SBL algorithm, the adaptive space-frequency structure prior model described by (8) is designed and applied in the SBL framework to reduce spatial aliasing and improve angular resolution. The model is based on the consideration of (i) the spatial statistics correlation when sources are densely distributed, and (ii) the frequency domain statistical correlation when the source is a wideband source or the sources have overlapping frequency bands. The prior can adaptively change to fit heterogeneous sources DOA scenarios according to the different structural patterns of the coefficients in the sparse signal. This change operation is realized by different elements in the correlation parameter matrices \(\nu\) and \(\upsilon\), which are determined based on the support matrix \(S\) of the space-frequency matrix \(X\).

The design of such an adaptive prior model means that it can change to fit scenarios with heterogeneous sources and has the following relationship with other SBL-based algorithms:

- When \(\upsilon_{l,f} = \nu\), the prior is similar to the joint sparsity in the frequency domain, and is suited to wideband DOA estimation of all sources that share the frequency band.
- When \(\upsilon_{l,f} = 0\) and \(\upsilon_{l,f} = 0\), the prior degenerates to the traditional SBL algorithm \(X_{l,f} \sim CN(0, \nu_{l,f}^\text{−1})\), which is suitable for single frequency and line spectrum signals.
- When \(\upsilon_{l,f} = \nu\) and \(\upsilon_{l,f} = 0\), the prior is the PC-SBL algorithm \(X_{l,f} \sim CN(0, (\nu_{l,f} + \nu_{l,f}^\text{−1})^\text{−1})\), which only considers spatial domain correlations and is suitable for block-sparse signals.

**B. The Computational Complexity**

The ASF-SBL algorithm uses VBI under the SBL framework to realize the heterogenous wideband sources DOA estimation. In the solution of the observation data at each frequency point, the main computational complexity of the SBL algorithm arises from the necessity to solve the variance \(\Sigma_{x,l}\) in (15), that is, the matrix inverse \((\alpha_0^\text{−1}\Phi_f^\text{−1} + \Phi_f^\text{−1})\). The computational complexity required for this inversion process is \(O(L^3)\). Since a reduction of computational complexity can be realized by describing the matrix inversion process as shown in (41), according to the Woodbury matrix identity

\[
\Sigma_{x,l} = \Lambda_{x,l} - \Lambda_{x,l} \Phi_f^\text{−1}(\alpha_0^\text{−1}I_M + \Phi_f^\text{−1}\Phi_f^\text{−1})^{-1}\Phi_f^\text{−1}\Lambda_{x,l} = \Lambda_{x,l} - \Lambda_{x,l} \Phi_f^\text{−1}\Sigma_{y,l}^\text{−1}\Phi_f^\text{−1}\Lambda_{x,l},
\] (41)

where \(\Sigma_{y,l} = \alpha_0^\text{−1}I_M + \Phi_f^\text{−1}\Phi_f^\text{−1}\). After simplification, only the inverse of the \(M\times M\)-dimensional matrix \(\Sigma_{y,l}\) is required, where \(L \gg M\). The computational complexity of the SBL algorithm is reduced from \(O(L^3)\) to \(O(M^3)\). Therefore, the main computational complexity of the ASF-SBL algorithm based on the SBL framework is \(O(M^3)\).
The traditional SBL algorithm [25], the PC-SBL algorithm [35], the RVM algorithm [31] and the FOCUSS algorithm [18] are also implemented, and their main computational complexity is $O(M^3)$ produced by matrix inversion. The main computational complexity of these algorithms is the same as that of the ASF-SBL algorithm. However, the computational complexity of the ASF-SBL algorithm is slightly higher than that of the aforementioned algorithms. This is because, in addition to matrix inversion, the algorithm also needs to calculate the support matrix $S$ and correlation parameter matrices $\nu$ and $\upsilon$. This higher complexity is compensated for by the improved performance of the algorithm, which is assessed using the numerical experiments discussed in Section VI.

References [48]–[51] provide that the complex-domain data is converted to the real-domain for processing, which can reduce the computational complexity. However, due to the need to calculate the real and imaginary parts of the data, the computation time for applying it to the ASF-SBL algorithm for one iterative update is similar. Through MATLAB simulation, one update time in the complex-domain is about 1.93 seconds, while the processing time in the real-domain is about 2.25 seconds.

VI. NUMERICAL EXPERIMENTS

A. 2-D DOA Simulation of Heterogeneous Wideband Sources

This section assesses the performance of the ASF-SBL algorithm for 2-D DOA estimation of heterogeneous wideband sources. In the simulation, the signal receiving device is a 19-elements cross array with an array element interval of 0.25 m. The array can be divided into two 10-elements ULAs. There are four incident sources:

- **source 1** with the DOA of $\{31^\circ, 49^\circ\}$ and frequency characteristics of 400:1:599 Hz;
- **source 2** with the DOA of $\{255^\circ, 21^\circ\}$ and frequency characteristics identical to source 1;
- **source 3** with the DOA of $\{151^\circ, 60^\circ\}$ and frequency characteristics of 640:20:820 Hz;
- **source 4** with the DOA of $\{299^\circ, 60^\circ\}$ and frequency characteristics of 580 Hz (single-frequency).

Note that: the nomenclature 400:1:599 Hz, for example, signifies that the frequency ranges from 400 Hz to 599 Hz with an interval of 1 Hz. Gaussian white noise is used to simulate the measurement noise with a SNR = 0 dB. The interval of the pre-defined grid is $2^\circ$.

Using the ASF-SBL algorithm for on-grid DOA estimation, the space-frequency map estimation results corresponding to the two ULA models shown in Fig. 6 can be obtained. The estimation results of space angle, DOA and frequency characteristics of each source obtained by the ASF-SBL algorithm are recorded in Table III. In addition, we also perform angle compensation on the on-grid estimation results and obtain the off-grid estimation results, which are also presented in Table III. Fig. 6 and Table III show that there are four sources in the entire spatial domain, and the estimation results of the DOA and frequency characteristics of these four sources are consistent with the true values. The results from the off-grid estimation are more accurate than that of the on-grid estimation. The increased accuracy for the off-grid estimation is because the true space angles do not fall on the grid.

B. Algorithm Performance Comparison

In this subsection, a comparison is given of the ASF-SBL algorithm, the traditional SBL algorithm [29], the PC-SBL algorithm [35], the RVM algorithm [31], the FOCUSS algorithm [18], and the Beamforming algorithm [52] for different source scenarios. For the convenience of analysis, the experiments only conduct 1-D DOA estimation.

![Fig. 6. Space-frequency map estimation results obtained from the ASF-SBL algorithm. Refer to Fig. 5 for the color map. There are total four sources in the spatial domain. Two sources have the same frequency characteristics. The other two sources are a single-frequency signal and a line spectrum signal, respectively.](image)

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>THE DOA AND FREQUENCY CHARACTERISTICS ESTIMATION RESULTS OF ASF-SBL ALGORITHM. IT CAN BE SEEN THAT THE OFF-GRID ESTIMATION RESULTS ARE MORE ACCURATE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>source 1</strong></td>
<td>True value</td>
</tr>
<tr>
<td>${\phi, \theta}$</td>
<td>$(91^\circ, 49^\circ)$</td>
</tr>
<tr>
<td>frequency</td>
<td>400:1:599Hz $^1$</td>
</tr>
<tr>
<td><strong>source 2</strong></td>
<td>${\phi, \theta}$</td>
</tr>
<tr>
<td>frequency</td>
<td>400:1:599Hz</td>
</tr>
<tr>
<td><strong>source 3</strong></td>
<td>${\phi, \theta}$</td>
</tr>
<tr>
<td>frequency</td>
<td>440:20:820Hz</td>
</tr>
<tr>
<td><strong>source 4</strong></td>
<td>${\phi, \theta}$</td>
</tr>
<tr>
<td>frequency</td>
<td>580Hz</td>
</tr>
</tbody>
</table>

$^1$This means that the frequency range from 400 Hz to 599 Hz at 1 Hz interval.
Fig. 7. The space-frequency map estimation results of different algorithms in different cases when the SNR is 0 dB, where (the 1-th row) case 1, (the 2-th row) case 2 and (the 3-th row) case 3 indicate that the frequency characteristics of two closely-spaced sources completely overlap, partially overlap and non-overlap, respectively. Refer to Fig. 5 for the color map. The ASF-SBL algorithm has better estimation results and a cleaner space-frequency map than the other algorithms, which is consistent with the comparison results in Fig. 8.

Fig. 8. RMSE of each algorithm under different SNR in different cases, where case 1, case 2 and case 3 are consistent with the cases in Fig. 7. The ASF-SBL algorithm has better performance than the other algorithms, which is consistent with the comparison results in Fig. 7.

Suppose that two sources are incident on a 10-elements ULA with the element interval of 0.25 m. The DOAs of source 1 and source 2 are 100° and 102°, respectively, and the interval of the pre-defined grid is 2° (the two sources are closely-spaced). For the simulation, the frequency characteristics of source 1 is 400:1:599 Hz and the frequency characteristics of source 2 is set to one of the following three cases: a) 600:1:799 Hz; b) 500:1:699 Hz; or c) 400:1:599 Hz. Gaussian noise is used to simulate the measurement noise and the SNR is 0 dB.

The space-frequency map estimation results for different scenarios are calculated using the ASF-SBL or the alternative algorithms and the results are shown in Fig. 7. Compared with the other algorithms, the ASF-SBL algorithm has better estimation results and a cleaner space-frequency map. The space-frequency map results obtained by the FOCUSS algorithm are more cluttered than those obtained by the other three comparison algorithms. Although the comparison algorithms can estimate the existence of sources at about 100° and 102°, the results at most frequency points are wrong. The space-frequency map obtained by the Beamforming algorithm cannot distinguish the two sources in the overlapping frequency band.

To investigate the effect of the SNR, let’s define the RMSE of DOA estimation of sources with different frequency characteristics in /Monte Carlo experiments as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{I} \sum_{k=1}^{K} \left( \sum_{f=1}^{F(k)} \left( \hat{\theta}_{k}^{(i,f)} - \theta_{k} \right)^2 \right)}{IK}},$$

where \(\hat{\theta}_{k}^{(i,f)}\) represents the DOA estimation result of the \(k\)-th source at the \(f\)-th frequency point in the \(i\)-th Monte Carlo experiment. \(\theta_{k}\) is the true DOA of the \(k\)-th source. \(F(k)\) is the number of frequency points occupied by the \(k\)-th source.

To assess the effect of the SNR in the above three scenarios, we set the SNR to increase from −5 dB to 20 dB at 5 dB intervals and perform 100 Monte Carlo experiments at each interval. Fig. 8 shows the RMSE for each experiment. Based on the RMSE metric, the ASF-SBL algorithm can obtain better estimation performance in different scenarios when the SNR is low. The Beamforming algorithm is better than the ASF-SBL algorithm in case 3. However, for closely-spaced sources with overlapping frequency bands, such as case 1 and case 2 (shown in Fig. 7), the Beamforming algorithm cannot distinguish the two sources. Figs. 8(a)-8(b) do not, therefore, show the RMSE of the Beamforming algorithm. The PC-SBL algorithm is better than the SLB algorithm and the RVM algorithm, which is mainly because the relationship between each coefficient and its neighboring coefficients in the spatial domain is considered in the former but not the latter. The wideband FOCUSS algorithm
has poor performance when SNR < 5 dB because the estimated space-frequency map is more cluttered at a lower SNR.

C. Helicopter’s DOA Estimation in the Presence of Interference

In this simulation, relatively pure helicopter audio with a fundamental frequency of 21 Hz is selected as the incident source, and this signal is incident on a 10-elements ULA with \( d = 0.25 \) m and a DOA of 60°. A line spectrum signal with the frequency characteristics of 40:20:220 Hz is used to simulate interference and its DOA is set equal to 130°. Part of the frequency band of the interference and the helicopter signal approximately overlap, which could make it impossible to accurately distinguish the two signals. The signal-to-interference ratio (SIR) is set to 0 dB. Gaussian noise is used to simulate the measurement noise and the SNR is set to 5 dB, −5 dB, or −15 dB to simulate scenarios where the distance between the helicopter and the array are different. We use the ASF-SBL algorithm and the high-resolution MUSIC algorithm for DOA estimation.

Fig. 9 shows the spatial spectrum results of the MUSIC algorithm. The spatial spectrum can be effectively estimated when the SNR is −5 dB and 5 dB, and there are sources at 60° and 130°. When the SNR drops to −15 dB, the algorithm cannot estimate the DOA. Although the spatial spectrum can be obtained with the SNR at −5 dB and 5 dB, the DOA of the helicopter and interference cannot be distinguished. The DOA 130° corresponding to the maximum spectrum peak is mistakenly recorded as the DOA of the helicopter.

Unlike the MUSIC algorithm, which cannot distinguish the DOAs of the two sources, the space-frequency map of the ASF-SBL algorithm shows the DOA and frequency characteristics of each source. The ASF-SBL algorithm can distinguish the DOAs of the helicopter and interference from the space-frequency map based on the frequency prior of the helicopter. Fig. 10 shows the space-frequency maps of the ASF-SBL algorithm with different SNRs. There are two sources in the entire airspace and, according to the frequency prior of the helicopter, the DOA results of the helicopter and interference are 60° and 130°, respectively. Fig. 11 shows the frequency domain amplitude estimation results of the helicopter at different SNRs extracted from the space-frequency map of Fig. 10. The ASF-SBL algorithm can accurately estimate the amplitude information of the helicopter at 5 dB and −5 dB, but there is a large error in the estimation result at −15 dB.
The low and high values of SNR are used to simulate situations where the helicopter signal is far from and near to the array, respectively. The results presented here show that the ASF-SBL algorithm is advantageous for long-distance direction-finding.

D. Solution of ASF-SBL Algorithm Under Non-Gaussian Signals

We verify the performance of the ASF-SBL algorithm under non-Gaussian signals. Two chirp signals are incident on a 10-elements ULA with an array element interval of 0.25 m. The DOAs of source 1 and source 2 are 100° and 130°, respectively. The frequency band range of source 1 is 400 Hz to 600 Hz, the frequency band range of source 2 is 500 Hz to 600 Hz. Alpha-stable distribution non-Gaussian noise with characteristics exponent $\vartheta = 1.2$ is set as measurement noise and the SNR is set as 0 dB and 5 dB, respectively. Note that the parameter $\vartheta$ is the characteristics exponent of Alpha-stable distribution noise and satisfies $\vartheta \in (0, 2)$. When $\vartheta$ is 1.2–1.6, it can represent most impulse noise found in the real environment.

Fig. 12 shows that the ASF-SBL algorithm can realize DOA estimation of non-Gaussian signals under non-Gaussian noise background. When the SNR is 0 dB, the DOA estimation results of the two sources at each frequency point fluctuate around 100° and 130°, respectively. When the SNR is 5 dB, the space-frequency map estimation result is purer than that when the SNR is 0 dB, and the DOA estimation results of sources at each frequency point are more accurate. This is mainly due to the fact that there are more extreme observations in the impulse noise under low SNR, which leads to the degradation of algorithm performance.

VII. CONCLUSION

In this paper, we propose the ASF-SBL algorithm to achieve DOA estimation of heterogeneous wideband sources with different frequency characteristics. Based on the spatial-frequency statistical correlation of the sparse signal, the ASF-SBL algorithm performs space-frequency joint processing on wideband signals. The joint processing reduces the spatial aliasing and improves the angular resolution. The space-frequency joint processing of the algorithm is realized by designing an adaptive space-frequency structure prior model and applying it to the SBL framework. The adaptive selection of the prior is determined from nine space-frequency structural patterns based on the space-frequency correlation of the sparse coefficient itself. The algorithm can, therefore, avoid the structural mismatch problem caused by the invariant prior of the traditional SBL-based DOA algorithms and is suitable for various DOA estimation scenarios.

In addition, we introduce a distributed processing method to extend the ASF-SBL algorithm to 2-D DOA estimation. The computational complexity is greatly reduced by decoupling the 2-D DOA estimation into two 1-D angle estimations. Numerical simulation results show that, the ASF-SBL algorithm is better than other algorithms, and its advantages are most pronounced for situations including low SNRs and closely-spaced sources.

REFERENCES


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