Novel Water-Filling for Maximum Throughput of Power Grid, MIMO, and Energy Harvesting Coexisting System With Mixed Constraints

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***Abstract*— Multiple-input multiple-output (MIMO) technology equips the transmitters with the multiple antennas. It can combine with energy harvesting (EH) to lift the spectrum efficiency and make use of a greener energy resource. A power grid is added to serve as a supplementary source to regulate the not-so-stable harvested energy supply of the system. Besides the MIMO technology being used, the power allocated to the user provided by both EH and the power grid is subject to the epoch power upper bound constraints. The background of these constraints comes from field requirements, such as avoiding the saturation of power allocated to the user(s), avoiding system-level out-of-band power leakage, and reducing interference with other transmitter(s) due to the non-linearity generated via the transmitting mechanisms to the user(s). The epoch power upper bound constraints make this problem more challenging, with the controllable power grid energy budget and its allocation. This paper applies our recently proposed geometric water-filling with group upper bounded power constraints and recursion machinery to form the proposed algorithm for solving the proposed throughput maximization problem. Our algorithm is precisely defined, and further provides the exact solution via the lower degree polynomial complexity. This point is very suitable for the massive MIMO system. To the best of our knowledge, no prior algorithm has been reported in the open literature to solve the targeted problem in this paper.**

***Index Terms*— Radio resource management, power grid (PG), multiple input multiple output (MIMO), energy harvesting (EH), water-filling algorithm with mixed constraints, exact solution.**

I. INTRODUCTION

With the increasing demand to energy supplies, the renewable energy sources have become an important alternative for the conventional fuel energy to protect environment. Energy harvesting (EH) techniques will have a major role in wireless communications, to look for "greener communications" [1].

There has been recent research effort on understanding data transmission in this kind of wireless communication systems with EH, for example, the investigation of power allocation policies [2]- [7], medium access control protocols [8], adaptive opportunistic routing protocols [9], [10], network throughput of a mobile ad hoc network powered by energy harvesting [11], and energy management in wireless sensor networks [6], [12], *etc*. In [13], optimality of a variant of the back pressure algorithm using energy queues is discussed. In [14], transmitters with energy harvesting and batteries of finite energy are considered and the problem of throughput maximization is solved offline in a (non-fading) static channel. As a fundamental work, which can provide an algorithm for transmission with energy harvesting, [5] investigated the throughput maximization problem with the given full side information and proposed some approaches making use of the water-filling approach to solve the Karush-Kuhn-Tucker (KKT) conditions [15] of the target problem. Due to the usage of bisection for the proposed water-filling algorithm, it cannot offer an exact solution within a finite amount of computation. Here, if an optimization algorithm can give the exact value of an optimal solution, such a solution is called the exact solution, contrasted with an approximate solution or point (refer to [16] and the references therein). For the fading channel cases, [17] proposed the "directional water-filling algorithm" scheme without a formal statement and the optimality proof of the proposed algorithm. All algorithms reported in these papers above are mainly innovative applications of water-filling mechanism for green communication, but they cannot compute the exact solutions, unlike ours.

With water-filling, more power is allocated to the channels with higher gains to maximize the sum of data rates of all the sub-channels [18]. The conventional way to solve the water-filling problem is to solve the KKT conditions, and then find the water-level(s) and the solutions. In [19], we proposed an approach from simple geometric meaning of water-filling (GWF). GWF has been extended to solving the problems with more complicated constraints (refer to [19]) which the conventional water-filling cannot solve. By recursively applying GWF and non-decreasing water-level condition, we proposed a recursive GWF (RGWF) in [20] to solve power allocation problem with energy harvesting transmission.

The wireless communication networks, equipped with the Multiple Input Multiple Output (MIMO) technology, have attracted extensive research attention to improve transmission rate and spectrum efficiency [21], [22]. In this paper, GWF and recursion machinery are *newly* applied to obtain the

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optimal power allocation solution to the throughput maximization problem of the Power Grid (PG), MIMO and EH coexisting system with mixed constraints. To design a simple method, which can however solve a very difficult problem, is just what we are looking for. Since the harvested energy in a current epoch cannot be used in its previous epochs, which is called causality constraint, this causality forms a family of sum power constraints for the harvested power. Also, PG has a budget or limit to distribute its power. This budget results in another sum power constraint for the power from PG. At the same time, the sum of power from both EH and PG is subject to the (sum) power upper bound constraint, for each of the epochs. These constraints are called the epoch power upper bound constraints in this paper. It forms another family of the upper bound (or peak) power constraints, for all the epochs. This family of the constraints comes from field requirements, such as avoiding the saturation of power allocated to the user(s), avoiding system level out-of-band power leakage, and reducing interference with other transmitter(s) due to the non-linearity generated via the transmitting mechanisms to the user(s). In this paper, the mixed constraints means the three families of the constraints together. Furthermore, the MIMO maximum throughput problems including the proposed problem are a class of complex-valued semi-definite optimization problems with a group of non-strict inequality constraints. Their objective or constraint functions are not all differentiable. Thus, there do not exist the KKT conditions, unlike those in the real spaces. Recent work in [23] raised a group of conditions for a stationary point. However, such a group of conditions cannot handle the class of complex-valued semi-definite optimization problems with a group of non-strict inequality constraints with inqualities (refer to [23, Ch. 6-7]). That is to say, [23] handles that with equality constraints, at most. In this regard, it is important to transform the complex valued optimization problem into an equivalent real valued problem. Its clear definition may refer to the one proposed in [24], as one of our earlier contributions.

If some of the mixed constraints are relaxed or some parameters take special values, the investigated problems regress into some special cases. For example, letting the epoch power upper bound constraints be large enough or be relaxed, and the number of the antennas be equal to one, the proposed general case is regressed into the individual case only consisting of SISO, EH and PG, as a coexisting system. The proposed algorithm can exactly solve the general problem, including various special cases. Some existing algorithms reported in the literature can solve special cases but cannot guarantee to be able to solve the target problem in this paper, *e.g.* [25]. In addition, [25] also offers an instance for necessity of studying the coexisting system and the off-line computation.

For the target problem, a recursive algorithm is proposed to compute the exact solution efficiently in this paper. This algorithm is referred to as PAMEC for the **P**ower **A**llocation for the maximum throughput of the PG, **M**IMO and **E**H Coexisting system with mixed constraints. Furthermore, for comparing with existing algorithms, we choose an efficient optimization algorithm with great promise: the primal-dual interior point method (PD-IPM) [26], as a comparison benchmark. PD-IPM, as the well known algorithm of IPM, has the advantage of the polynomial computational complexity over other existing convex optimization methods, even though it only guarantees an ϵ solution, not optimal (refer to [26] and the references therein), and only is able to be used in a real space. Thus, for our proposed problem, PD-IPM has to use our constructed real equivalent model to compare. As a side note, *under merit of the exact solution and the polynomial complexity, there is no existing algorithm as comparison reference, to the best of the authors' knowledge.* Compared with our previous works, such as [27]- [31], they only either handle some special cases or use much complicated procedures for the EH and power grid cases with the epoch power upper bound constraints. However, they cannot compute the exact solution to the proposed more general problem, unlike this paper. This is due to the different machinery being used in this paper that the allocation process comes from the final epoch successively back to the first epoch over the interested time window. It makes the procedures much simpler, although the target problem to solve is more general. Furthermore, the conference papers are also short of the details due to space limitation. *In summary*, the proposed new water-filling owns two distinct advantages: simple and elegant, and efficient. The "simple and elegant" feature refers to the fact that the algorithm is clearly and easily implemented to solve a very complicated problem; and the "efficient" feature means another fact that the proposed algorithm computes the exact solution with a polynomial computational complexity.

In the remaining of the paper, system description and problem statement are presented in Section II. The proposed power allocation algorithm for maximum throughput is investigated in Section III. Numerical examples and complexity analysis are presented in Section IV. Section V concludes the paper.

Key notations that are used in this paper are as follows: $|\mathbf{A}|$ and $\mathrm{Tr}\,(\mathbf{A})$ give the determinant and the trace of a square matrix $\mathbf{A}$, respectively; $E[X]$ is the expectation of the random variable X; and the capital symbol $\mathbf{I}$ for a matrix denotes the identity matrix with the corresponding size. A square matrix $\mathbf{B} \succeq 0$ means that $\mathbf{B}$ is a positive semi-definite matrix. Further, for arbitrary two positive semi-definite matrices $\mathbf{B}$ and $\mathbf{C}$, the expression $\mathbf{B} \succeq \mathbf{C}$ means the difference of $\mathbf{B} - \mathbf{C}$ is a positive semi-definite matrix. In addition, for any complex matrix, its superscripts $\dagger$ and T denote the conjugate transpose and the transpose of the matrix, respectively.

II. System Description and Problem Statement

In this section, energy harvesting in a fading channel is presented, followed by the optimization problem to maximize the throughput, for the PG, MIMO and EH coexisting system with the mixed constraints, and then by its real equivalence form. For convenience and without loss of generality, the process is assumed to be a discrete time process.

A. System Description

As shown in Fig. 1, the system model depicts the time period from $(0, T]$ including K epochs. At the beginning of

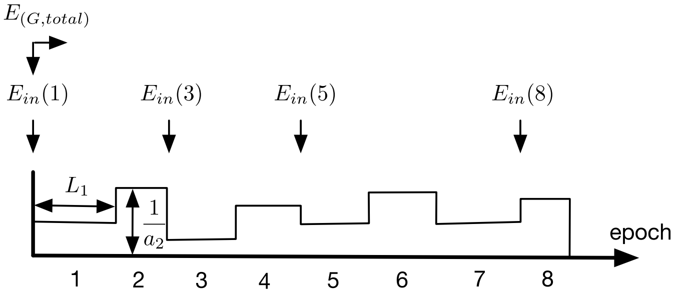

Fig. 1. System description, $K = 8$ epochs in $(0, T]$.

each epoch, we can observe the fading gain change or the harvested energy arrival, or both. Let L_k and a_k denote the time duration and the fading channel gain of the kth epoch, where $k = 1, \ldots, K$. Without loss of generality, assume $L_k > 0, a_k > 0, \forall k$. At the beginning of the kth epoch, the harvested energy is denoted by $E_{in}(k)$, and the event of energy harvesting is depicted as $E_{in}(k) > 0$. We assume that $\overline{P}_k \geq 0$, as the epoch power upper bound constraint on the sum power from PG and EH for the kth epoch, $\forall k$. For all the K epochs, the budget $E_{(G,\text{total})}$ of power, from PG, expresses that the sum of all the power provided by PG cannot be beyond $E_{(G,\text{total})}$. As a side note, due to the causality, $E_{(G,\text{total})}$ can be combined with $E_{in}(1)$ at the beginning of the first epoch in Fig. 1, equivalently, but it has a different source from $\{E_{in}(k)\}$. Since our purpose is not only to optimize performance of the throughput but also to guarantee the optimal power allocation solution to meet the three families of the mixed constraints. This paper investigates the cases of both the multiple antennas, and the PG and EH coexisting with the mixed constraints, as a whole to differentiate with previous works. Thus, although the proposed problem is an optimization problem, its optimization variables are complex matrix-valued. There exists the differentiability issue of the objective function. As a result, direct or superficial usage of the KKT conditions [26] cannot guarantee a correct result. It is simple but important to transform the proposed problem into an equivalent optimization problem with real optimization variables.

B. Problem Statement

For the proposed energy source management system, assume that the optimal power management strategy is such that the transmit power is constant in each epoch. This is because the discussed process is assumed to be discrete in the index variable of the epochs. Therefore, let us denote the transmit power at the kth epoch be $\text{Tr}(\mathbf{S}_{H,k})$ $(k = 1, \ldots, K)$, where $\mathbf{S}_{H,k} = E[\mathbf{s}_{H,k}\mathbf{s}_{H,k}^{\dagger}]$ and $\mathbf{s}_{H,k} \in \mathbb{C}^{N_t \times 1}, \forall k$, is the part, utilizing the power from energy harvesting, of the transmitted complex vector-valued signals by the user equipped with N_t transmit antennas. Similarly, $\mathbf{S}_{G,k} = E[\mathbf{s}_{G,k}\mathbf{s}_{G,k}^{\dagger}]$ and $\mathbf{s}_{G,k} \in \mathbb{C}^{N_t \times 1}, \forall k$, is the part, utilizing the power from PG, of the transmitted complex vector-valued signals by the user. The receiver side of the transceiver is equipped with N_r antennas. The channel matrix at epoch k is denoted $\mathbf{H}_k \in \mathbb{C}^{N_t \times N_R}, \forall k$, *i.e.*, $\mathbf{y}_k = \mathbf{H}_k^{\dagger}(\mathbf{s}_{H,k} + \mathbf{s}_{G,k}) + \mathbf{z}$, where $\mathbf{z}$ is an additive Gaussian noise. For this MIMO system, details of the assumptions above can further refer to those in [32]. The objective is to maximize the total throughput by the deadline T, *i.e.*, within the K epochs. We have mentioned the causality that current harvested energy cannot be used by the previous epochs but it can be used by the following epochs. We further assume that $E_{\max}$, the battery capacity constraint is relaxed, *i.e.*, assuming $E_{\max} \gg 0$. Hence, the optimization problem in this PG, MIMO and EH coexisting system with the mixed constraints can be written as:

$$\begin{aligned} \max_{\{\mathbf{S}_k\}_{k=1}^{K}} \ & \sum_{k=1}^{K} \frac{L_k}{2} \log |\mathbf{I} + \mathbf{H}_k^{\dagger}(\mathbf{S}_{H,k} + \mathbf{S}_{G,k})\mathbf{H}_k| \\ \text{Subject to: } & \mathbf{S}_{H,k} \succeq 0, \mathbf{S}_{G,k} \succeq 0, \ \forall k; \\ & \text{Tr}(\mathbf{S}_{H,k} + \mathbf{S}_{G,k}) \leq \overline{P}_k, \ \forall k; \\ & \sum_{k=1}^{l} L_k \text{Tr}(\mathbf{S}_{H,k}) \leq \sum_{k=1}^{l} E_{in}(k), \\ & \text{for } l = 1, \ldots, K; \\ & \sum_{k=1}^{K} L_k \text{Tr}(\mathbf{S}_{G,k}) \leq E_{(G,\text{total})}, \end{aligned} \tag{1}$$

where the first two sets of the constraints account for the guaranteed nonnegative allocated power, and the epoch power upper bound constraints. The third set of the constraints accounts for the causality. The fourth set of the constraints accounts for the budget of sum power from PG. In addition, it is seen that the epoch power upper bound constraints on the sum of both the harvested power and the power from PG for every epoch, determines the second set of the constraints. This point and the optimal allocation of the two classes of power result in the proposed problem having a difficult form so that no prior algorithm in the open literature can solve the proposed problem under the merit of exactness and efficiency. The stated discrete process leads to the discrete time optimization problem of (1). It is well known that, for any index k of the epochs, the optimization variables $\{\mathbf{S}_{H,k}, \mathbf{S}_{G,k}\}$ of (1) are not assumed into a point in a function space, but they are a point in $\mathbb{C}^{N_t \times N_t}$. This point just explains that "the transmit power is constant in each epoch".

The investigation in this paper also lays down a foundation to solve the more challenged cases of finite E_{max} and then carry out a real-time ("on-line") computation.

C. Real Equivalence Form of the Target Problem

Due to the objective function of (1) in several complex optimization variables, existing optimization methods cannot directly apply to (1). A real equivalence form of (1) will be proposed. For discussing equivalence between these two forms, the formal definition of two optimization problems being equivalent [24] is revisited.

Definition 1 (Equivalence Between Two Optimization Problems): Two optimization problems are said to be equivalent iff there exists a bijection between their optimal solution sets.

A real optimization problem is easily obtained, which is mentioned below and equivalent to the target problem: (1). This equivalent real form of the target problem is:

$$\begin{aligned}
&\max_{\{s_{H,i},s_{G,i}\}_{i=1}^{N_t\times K}} \sum_{i=1}^{N_t\times K} w_{[\frac{i-1}{N_t}]+1}\log(1+a_i s_i)\\
&\text{subject to: } s_i = s_{H,i}+s_{G,i},\ \forall i;\\
&\qquad 0\le s_{H,i}, 0\le s_{G,i},\ \forall i;\\
&\qquad \sum_{i=(l-1)\times N_t+1}^{l\times N_t} s_i \le \overline{P}_l,\\
&\qquad \text{for } l=1,\ldots,K;\\
&\qquad \sum_{i=1}^{l\times N_t} s_{H,i} \le \sum_{k=1}^{l} E_{in}(k), \forall l;\\
&\qquad \sum_{i=1}^{K\times N_t} s_{G,i} \le E_{(G,\text{total})},
\end{aligned} \tag{2}$$

where $\frac{a_i}{L_{[\frac{i-1}{N_t}]+1}} \rightarrow a_i$, $L_{[\frac{i-1}{N_t}]+1}s_{(H,\text{ or }G,)i} \rightarrow s_{(H,\text{ or }G,)i}$, $\frac{L_l}{2} \rightarrow w_l$, $L_l\overline{P}_l \rightarrow \overline{P}_l$, for any i, l, and [ ] denotes the integral part of a real number. For simplicity, we write $\{w_{[\frac{i-1}{N_t}]+1}\}$ as $\{w_i\}$. Note that the symbol "$\rightarrow$" is the assignment operator. Without consideration of trivial cases, $\overline{P}_k > 0$ and $E_{in}(i) > 0$ can be assumed. Equivalence between (1) and (2) can be stated by the following lemma, the proof of which contains the bijection mentioned in the definition. In essence, except the change of variables: $s_i = s_{H,i}+s_{G,i},\ \forall i$, (2) has the same four sets of constraints as (1). As a side note, the channel gains $\{a_i\}_{i=(k-1)N_t+1}^{kN_t}$ correspond to the multiple antennas equipped by the transmitter, where the variable k corresponds to the index of the kth epoch $\forall k$, *i.e.*, setting up the MIMO channels at epoch k. Also, in many applications, such gains are already sorted. This is because they come from the eigenvalues of a matrix and many of the algorithms to compute the eigenvalues and eigenvectors already produce the eigenvalues sorted [16], for any k.

Lemma 1: The optimization problems (1) and (2) are equivalent.

Proof: Resorting to the well known Hadamard's inequality on positive definite matrices, the equivalence may be proven. Given a family of the positive semi-definite matrices: $\{\mathbf{H}_k\mathbf{H}_k^\dagger\}_{k=1}^K$, there exists a family of the unitary matrices: $\{\mathbf{U}_k\}_{k=1}^K$ such that $\mathbf{U}_k^\dagger\mathbf{H}_k\mathbf{H}_k^\dagger\mathbf{U}_k$ is a diagonal matrix, denoted by $\mathbf{D}_k$, through the eigendecomposition, for $k = 1,\ldots,K$. Further, the major diagonal entries of this diagonal matrix are the eigenvalues of $\mathbf{H}_k\mathbf{H}_k^\dagger$. That is to say, the diagonal matrix and $\mathbf{H}_k\mathbf{H}_k^\dagger$ are unitarily similar. For convenience but without operations of the indexes, as an alternative notation, $(s_k(1),\cdots,s_k(N_t))$ denotes $(s_{(k-1)N_t+1},\cdots,s_{kN_t}), \forall k$, in (2). Thus, we have a bijection from the set of feasible solutions to (2) to that to (1): $(s_k(1),\cdots,s_k(N_t)) \mapsto \mathbf{U}_k\text{diag}(s_k(1),\cdots,s_k(N_t))\mathbf{U}_k^\dagger, \forall k$. The following will prove that the bijection is the aforementioned one in the definition of equivalence between two optimization problems. Using a similar method to that in [24], the equivalence is obtained. □

If the solution to the real optimization problem (2) can be computed rapidly and exactly, and a bijection has been constructed, (1) can be solved rapidly and exactly as well. As a side note, the optimization variables of (2) can be grouped, with letting $\chi_k = \{(k-1)N_t+1,\ldots,kN_t\}, \forall k$, for the index set of $(s_{(k-1)N_t+1},\cdots,s_{kN_t})$. Thus, $\{\chi_k\}_{k=1}^K$ is a partition of the index set of $\{a_i\}_{i=1}^{KN_t}$ in (2), denoting the indexes related to the kth epoch.

To find the solution to problem (2), we often solve a nonlinear system, including both nonlinear equations and inequalities, in the dual variables. This is because the conventional water-filling approach starts to obtain the Karush-Kuhn-Tucker (KKT) conditions of problem (2) as a set of optimality conditions, and then to solve this set of conditions to determine the optimal variables $\{s_{H,i}, s_{G,i}\}$ and their dual variables. For this set of KKT conditions, due to its complexity from the PG, MIMO, and EH coexisting system with the mixed constraints, there is no existing method available in the open literature to obtain an exact solution through a finite amount of computation. However, in this paper, we investigate the problem and propose an exact solution through a finite amount of computation.

Remark 1: The above content has accounted for the equivalence between (1) and (2). As a supplement, how to obtain (2) based on (1) is explained as follows. To obtain (2), it only needs to obtained $\{w_i\}$ and $\{a_i\}$ in (2) with respect to all the other parameters in (2) being able to come from (1). In the adjective clause in the sentence including (2), it utilizes the four assignment statements. Thus, to obtain $\{w_i\}$ and $\{a_i\}$ in (2), it only requires to obtain $\{a_i\}$ on the left hand side of the first assignment statement. Given $\{\mathbf{H}_k\mathbf{H}_k^\dagger\}_{k=1}^K$, the second sentence in the proof of Lemma 1 has provided the computation method for a family of the unitary matrices: $\{\mathbf{U}_k\}_{k=1}^K$. Therefore, the $\{a_i\}$ on the left hand side of the first assignment statement, mentioned above, can be obtained by $(a_{(k-1)N_t+1},\ldots,a_{kN_t})^T = \text{diag}(\mathbf{U}_k^\dagger\mathbf{H}_k\mathbf{H}_k^\dagger\mathbf{U}_k)$, for $k = 1,\ldots,K$. As a side note, $\mathbf{U}_k^\dagger\mathbf{H}_k\mathbf{H}_k^\dagger\mathbf{U}_k$ is a diagonal matrix and $(a_{(k-1)N_t+1},\ldots,a_{kN_t})^T$ takes the major diagonal of the diagonal matrix as its entries.

In the next section, the proposed algorithm is introduced to effectively solve problem (2). As a by-product, it also solves the KKT conditions based on our geometric approach, by constructing a set of the optimal dual variables.

III. Proposed Power Allocation Algorithm (PAMEC) to Solve (2)

In this section, we first introduce a functional block of PAMEC, as preparation. Then, PAMEC and its optimality are investigated.

A. Preparation of PAMEC

Since PAMEC uses the algorithm: Geometric Water Filling with Group Upper bound Power constraints (GWFGUP), as a fundamental block, GWFGUP will be introduced in this subsection. In fact, [19] has presented the algorithm of Geometric Water-Filling Group Bounded Power constraints (GWFGBP) for a class of the optimal radio resource management problems with the group bounded power constraints. Like the assumption in [19], for convenience of statement, GWFGUP in this paper means two things: a class of the optimal radio resource management problems with the group upper bound power constraints; and the algorithm to compute the exact solution to such a class of the problems. If the group lower bounded power constraints of GWFGBP are relaxed, then GWFGUP, as an individual case, is formed. Since PAMEC in this paper mainly uses GWFGUP for its fundamental block, GWFGUP, as the problem and the algorithm, is concisely introduced below respectively. Before this introduction, some notations need to be presented here: given $P \geq 0$, as the total power of the users or volume of the water. The allocated power, the propagation path gain, and the weight for the ith user are given as s_i, a_i and $w_i (\geq 0)$ respectively, $i = 1, ..., KN_t$, which has the index partition $\{\chi_k\}_{k=1}^{K}$; and $\overline{P}_k$, $1 \leq k \leq K$, is the upper bound for the sum of the total power by the kth group.

GWFGUP problem, as the throughput maximization problem, is:

$$\begin{aligned} \max_{\{s_i\}_{i=1}^{KN_t}} \quad & \sum_{i=1}^{KN_t} w_i \log(1 + a_i s_i) \\ \text{subject to: } & 0 \leq s_i, \ \forall i; \\ & \sum_{i=1}^{KN_t} s_i \leq P; \\ & \sum_{i \in \chi_k} s_i \leq \overline{P}_k, k = 1, \ldots, K. \end{aligned} \tag{3}$$

As a side note, it is assumed that the sequence $\{a_i w_i\}_{i=1}^{KN_t}$ is monotonically deceasing (since the indexes can be arbitrarily renumbered to satisfy this condition, noting all the members of χ_k also taking the corresponding change).

Then, for introducing GWFGUP, as an algorithm, the results for GWF (Geometric Water-filling) [19] are summarized below. GWF aims to solve the same problem as (3) with relaxation of the group upper bound power constraint, *i.e.*, $\overline{P}_k, \forall k$, is large enough. The water level step index i^* is obtained by [19]

$$i^* = \max\left\{ i \,\middle|\, P_2(i) > 0, \quad 1 \leq i \leq KN_t \text{ and } i \in E \right\}, \tag{4}$$

where E is an index set that is a subset of $\{1, \ldots, KN_t\}$ with the note that the set of $\{1, \ldots, KN_t\}$ is also a subset of itself, and $P_2(i)$ is a middle variable and given as

$$P_2(i) = \left[P - \sum_{j \in \{1 \leq j \leq i-1, j \in E\}} \left(\frac{1}{a_i w_i} - \frac{1}{a_j w_j} \right) w_j \right]^+, \quad \text{for } 1 \leq i \leq KN_t \text{ and } i \in E. \tag{5}$$

Then the power allocated for the i^* step is

$$s_{i^*} = \frac{w_{i^*}}{\sum_{j \in \{1 \leq j \leq i^*, j \in E\}} w_j} P_2(i^*). \tag{6}$$

The completed solution is then

$$\begin{cases} s_i = \left[\dfrac{s_{i^*}}{w_{i^*}} + \left(\frac{1}{a_{i^*} w_{i^*}} - \frac{1}{a_i w_i} \right) \right] w_i, \\ 1 \leq i \leq i^* \text{ and } i \in E; \\ s_i = 0, \quad i^* < i \leq KN_t \text{ and } i \in E. \end{cases} \tag{7}$$

(4)-(7) is illustrated by Fig. 2, as an intuitive explanation, to help better understanding GWF and GWFGUP. Figs. 2(a)-(d) give an illustration of (4)-(7). Suppose there are 4 steps/stairs inside a water tank. For the conventional approach, the dashed horizontal line, which is the water level μ, needs to be determined first and then the power allocated (water volume) above the step is solved. Let us use w_i to denote the width of the ith step. For the ith step, the allocated power s_i represents the area from the step to the surface of the water (if this step is under water). The term $d_i = \frac{1}{a_i w_i}$ represents the height from the step to the bottom of the tank. Instead of trying to determine the water level μ, which is a non-negative real number, we aim to determine water level step, which is an integer number from 1 to KN_t, denoted by i^*, as the highest step under water. Based on the result of i^*, we can write out the solutions for power allocation instantly. Fig. 2(a) illustrates the concept of i^*. Since the third level is the highest level under water, we have $i^* = 3$. The shadowed area denotes the allocated power for the third step by s_{i^*}. In the following, $P_2(i)$, the water volume above step i, can be obtained considering the step depth difference and the width of the stairs. As an example in Fig. 2(c), the water volume above step 1 and below step 3 with the width w_1 can be found as: the step depth difference, $(d_3 - d_1)$ multiplying the width of the step, w_1. Therefore, the corresponding $P_2(i = 3)$ can be expressed as

$$P_2(i = 3) = P - (d_3 - d_1) \cdot w_1 - (d_3 - d_2) \cdot w_2$$

illustrated by the shadowed area in Fig. 2(c). The weights as the widths are emphasized in Fig. 2(d).

In summary, for (4)-(7) being used by GWF, the first step is to calculate $P_2(i)$, then find the water level step, i^*, from (4), which is the maximal index making $P_2(i)$ positive. The corresponding power level for this step, s_{i^*}, can be obtained by applying (6). Then for those steps with index higher than i^*, the powers are assigned with zeros. For those

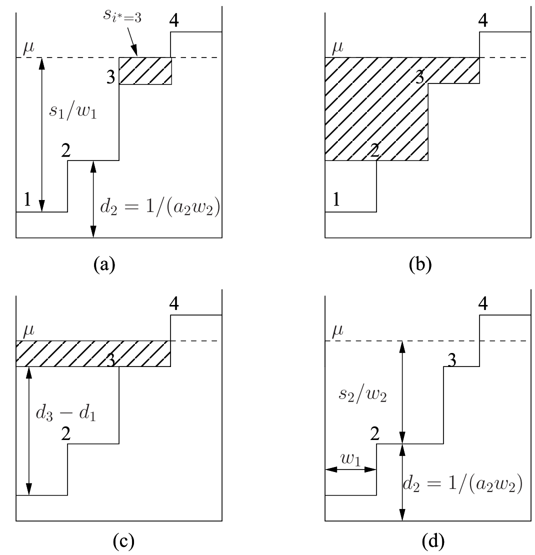

Fig. 2. Illustration of Concepts in (4)-(7) by GWF. (a) Illustration of water level step $i^* = 3$, allocated power for the third step s_{i^*}, and step/stair depth $d_i = \frac{1}{a_i w_i}$. (b) Illustration of $P_2(i)$ (shadowed area, representing the total water/power above step i) when $i = 2$. (c) Illustration of $P_2(i)$ when $i = 3$. (d) Illustration of the weights as the widths.

Algorithm 1 Pseudocode for GWFGUP

1: Input: $\{d_i = \frac{1}{a_i w_i}, w_i\}_{i=1}^{KN_t}$, $\{\overline{P}_k\}_{k=1}^{K}$, the set $E = \{1, 2, \ldots, KN_t\}$, and P;
2: Utilize (4)-(7) to compute $\{s_i\}$;
3: The set Λ is defined by the set $\{k | \sum_{j \in \chi_k} s_j > \overline{P}_k, k \in \{1, 2, \ldots, K\}\}$;
4: **if** Λ is the empty set **then**
5: Output $\{s_i\}_{i=1}^{KN_t}$;
6: **end if**
7: **if** Λ is not the empty set **then**
8: Let $\sum_{i \in \chi_k} s_i = \overline{P}_k$, as $k \in \Lambda$;
9: Utilize similar Eqns. (4)-(7) as these similar expressions. Note that these similar expressions differ only by replacing the set of E in (4)-(7) with the set of χ_k, for $k \in \Lambda$.
10: **end if**
11: Update E with $E \setminus (\cup_{k \in \Lambda} \chi_k)$, and P with $P - \sum_{k \in \Lambda} \overline{P}_k$;
12: Then return to 2) of GWFGUP.

steps below i^*, the powers are assigned as in (7). The first term (s_{i^*}/w_{i^*}) inside the square bracket denotes the depth of the i^*th step to the surface of the water. The second term inside the square bracket denotes the step depth difference of the i^*th step and the ith step. Therefore, the sum inside the square bracket means the depth of the ith step to the surface of the water. When this quantity is multiplied with the width of this step, the area of the water above this step (allocated power) can be then readily obtained.

Equipped with GWF algorithm, GWFGUP is formally listed in Algorithm 1.

As a side note, the optimality proof of Algorithm GWFGUP can refer to that of GWFGBP in [19].

Remark 2: Algorithm GWFGUP is a dynamic power distribution process. The state of this process is the difference, denoted by the set Λ, between the group upper bound power sequence and the current power distribution sequence obtained by (4)-(7). The control of this process is to mainly use the two following if-then statements in GWFGUP. A new state is updated in the next time stage. An optimal dynamic power distribution process through GWFGUP with the state feedback is thus formed. Since the finite set is getting smaller and smaller until the set E is empty, Algorithm GWFGUP carries out the K loops that updates the set E, at most. Then it obtains the optimal solution.

For a slight change of the discussion above, let L be a positive integer and $L \leq K$, where L denotes the index of the starting group of the channels for an entire process. Then this process is more general. Pointing to the target problem (2), it has been seen that $|\chi_i| = N_t$. Therefore, Algorithm GWFGUP can be regarded as a mapping from the point of parameters $\{L, K, \{w_i, a_i\}_{i \in \cup_{L \leq k \leq K} \chi_k}, P, \{\overline{P}_k\}_{k=L}^{K}\}$ to the solution $\{s_i\}_{i \in \cup_{L \leq k \leq K} \chi_k}$. That is to say, it can be written as a formal expression, similar to that in [19]:

$$\{s_i\}_{i \in \cup_{L \leq k \leq K} \chi_k} = \text{GWFGUP}\left(L, K, \{w_i, a_i\}_{i \in \cup_{L \leq k \leq K} \chi_k}, P, \{\overline{P}_k\}_{k=L}^{K}\right). \quad (8)$$

Algorithm 2 Pseudocode for PAMEC(K)

Input:
$K, \{E_{in}(k), \overline{P}_k\}_{k=1}^{K}, E_{(G,\text{total})}, \{s_{H,i} = 0, w_i, a_i\}_{i=1}^{KN_t}$;
for $L = K : -1 : 1$ **do**
$\{\triangle s_i\}_{i \in \cup_{L \leq k \leq K} \chi_k} =$ GWFGUP$(L, K, E_{in}(L), \{\overline{P}_k\}_{k=L}^{K})$;
for $k = L : 1 : K$ **do**
$a_i = \frac{a_i}{1 + a_i \triangle s_i}$, for $i \in \chi_k$;
$\overline{P}_k = \overline{P}_k - \sum_{i \in \chi_k} \triangle s_i$;
$\{s_{H,i} = s_{H,i} + \triangle s_i\}_{i \in \chi_k}$;
end for
end for
$\{s_{G,i}\}_{i=1}^{KN_t} =$ GWFGUP$(1, K, E_{(G,\text{total})}, \{\overline{P}_k\}_{k=1}^{K})$;
Output PAMEC(K): $\{s_{H,i}, s_{G,i}\}_{i=1}^{KN_t}$.

Note that, for conciseness and without confusion from context, we may write the right hand side of the expression as GWFGUP$(L, K, P, \{\overline{P}_k\})$ to emphasize time stages from L to K.

Since GWFGUP vividly uses water-filling to handle the throughput maximization problem with the power group upper bounds, it can be called water-filling with the power group upper bounds. Thus, PAMEC can be called water-filling for the PG, MIMO and EH coexisting system with the mixed constraints, mainly for "greener communications", as an alternative. Since PAMEC depends on the final index of the epochs: K, sometimes to emphasize this dependence, we also use PAMEC(K) to denote PAMEC. The pseudocode of the proposed PAMEC(K) is stated at the following subsection. Then, optimality of the proposed algorithm is also discussed at the following subsection.

B. Algorithmic Statement of PAMEC and Its Optimality

In this subsection, we discuss the power allocation for problem (2). The steps of the proposed PAMEC is listed in the pseudocode of Algorithm 2. Note that, for problem (2), it is seen from its KKT conditions, that the water levels of the epochs do not monotonically increase as the index of the epochs increases due to the group upper bounded power constraints. Along this way that through directly solving the KKT conditions to compute the optimal solution, it is rather difficult if it is not impossible. However, our solution is based on geometry, not directly solving the KKT conditions.

The proposed PAMEC first distributes the harvested energy, starting from the last epoch to apply GWFGUP, recursively to the first epoch. The allocated power is equivalently treated as increased step height inside the tank. After the allocation of the harvest energy, the energy from the grid is allocated by applying GWFGUP once. The procedure will be illustrated step by step when Example 1 is presented in Numerical Results Section.

Optimality of the proposed PAMEC(K) is stated by the following Proposition 1. Prior to setting up the optimality proof of the proposed PAMEC(K), or introducing Proposition 1, one lemma has been introduced above, and three lemmas need to be introduced next. As a side note, the statement of the proposed algorithm PAMEC(K) is clear and short, but the optimality proof of the proposed algorithm PAMEC(K) is rather difficult. For convenience, the logical line of the optimality proof is concisely stated here. Lemma 2 proposes optimality of PAMEC(K), for the individual case with $E_{(G,\text{total})} = 0$ in the target problem (1) or (2). This individual case corresponds to Lines 2-9 in the statement of Algorithm PAMEC(K). Then an implied problem by the target problem is introduced. Lemmas 3 and 4 reveal the relationship between the solution to the individual case with $E_{(G,\text{total})} = 0$ and that to the implied problem. As a naturally developed result of the four lemmas, Proposition 1 is obtained. Now, this development is unfolded next.

Lemma 2: PAMEC(K) can compute the exact solution to the individual case of problem (2), within finite loops, that lets $E_{(G,\text{total})} = 0$.

Proof: See Appendix A. □

It is seen that the target problem (2) implies the following optimization:

$$\begin{aligned} \max_{\{s_i\}_{i=1}^{KN_t}} \quad & \sum_{i=1}^{KN_t} w_i \log(1 + a_i s_i) \\ \text{subject to: } & 0 \le s_i, \ \forall i; \\ & \sum_{i \in \chi_k} s_i \le \overline{P}_k, 1 \le k \le K; \\ & \sum_{k=1}^{l} \sum_{i \in \chi_k} s_i \le E_{(G,\text{total})} + \sum_{k=1}^{l} E_{in}(k), \\ & l = 1, \ldots, K. \end{aligned} \tag{9}$$

The introduction of this implied optimization problem is motivated by two aspects: it can be utilized, for finally solving the target problem (2), with letting $s_i = s_{H,i} + s_{G,i}, \forall i$; and its exact solution can be efficiently computed by the same algorithm PAMEC(K) in Lemma 2, only via adding $E_{(G,\text{total})}$ into the harvested energy $E_{in}(1)$ for epoch 1 of (9). It is interesting that by solving the implied optimization problem, we can compute an optimal solution to the original problem. However, To establish this, other two lemmas are needed. Lemma 3 is proposed to offer a relationship between the solution to problem (2) and another solution to the implied optimization problem. Successively, Lemma 4 claims a relationship between the difference of the two aforementioned solutions and $E_{(G,\text{total})}$. Finally, an optimal solution to the target problem (2) will be proposed via Proposition 1.

Lemma 3: Let the solution to (9) be denoted by $\{\overline{s}_i^*\}_{k=1}^{KN_t}$, and let the solution to (2) with $E_{(G,\text{total})} = 0$ be denoted by $\{s_{H,i}^*\}_{1=1}^{KN_t}$. Then $\overline{s}_i^* \ge s_{H,i}^*, \forall i$.

Proof: See Appendix B. □

As an emphasis, it has been seen that PAMEC(K) can compute the exact solution to (9), as the implied problem of target problem (2); while PAMEC(K) can also compute the exact solution to problem (2) with $E_{(G,\text{total})} = 0$; and each member or entry of the exact solution to (9) is greater than the corresponding member of the exact solution to problem (2) with $E_{(G,\text{total})} = 0$. This emphasis is based on Lemmas 1-3. In fact, the aforementioned two exact solutions also satisfy the following property. Before state this property, as a reminder, problem (2) with $E_{(G,\text{total})} = 0$ and the implied (9) are all convex optimization problems with their strictly concave objective functions, respectively. Thus, optimal solutions to the two problems satisfy uniqueness. This point implies that any given solution to each of these two problems is just represented by the constructed one by the lemmas.

Lemma 4: Let the solution, to the implied (9), be denoted by $\{\overline{s}_1^*\}_{k=1}^{KN_t}$, and the solution, to problem (2) with $E_{(G,\text{total})} = 0$, be denoted by $\{s_{H,i}^*\}_{i=1}^{KN_t}$. Then $\sum_{i=1}^{KN_t}(\overline{s}_i^* - s_{H,i}^*) \le E_{(G,\text{total})}$.

Proof: See Appendix C. □

Remark 3: For clarity, motivations to propose the lemmas and the following proposition are stated here. Lemma 2 accounts for computing the optimal solution $\{s_{H,i}^*\}_{i=1}^{KN_t}$ to problem (2) with $E_{(G,\text{total})} = 0$, by lines 2-9 in the statement of PAMEC(K). Lemma 3 explains line 10 in the statement of PAMEC(K), together with $\{s_{H,i}^*\}_{i=1}^{KN_t}$, can determine the solution $\{\overline{s}_i^*\}_{i=1}^{KN_t}$ to the implied problem. Lemmas 3-4 propose the fact that the difference between $\{\overline{s}_i^*\}$ and $\{s_{H,i}^*\}$ satisfy the power constraints of PG. Further, the following proposition can offer the result that $\{s_{H,i}^*\}$ and $\{\overline{s}_i^* - s_{H,i}^*\}$ are an optimal solution to the target problem (2), where $\{\overline{s}_i^* - s_{H,i}^*\}$ is denoted by $\{s_{G,i}^*\}$.

Proposition 1: PAMEC(K) can exactly compute the optimal solution to problem (2) within finite loops. In addition, PAMEC(K) most efficiently utilizes the harvested energy, as the first priority of the (power) allocation.

Proof: First, assume $\{s_{H,i}^*, s_{G,i}^*\}_{i=1}^{KN_t}$ to be an optimal solution to problem (2). The optimal value of problem (2) has the same optimal value as that of problem (9). The reason is concisely stated as follows. Let $s_i = s_{H,i}^* + s_{G,i}^*, \forall i$. Thus, $\{s_i\}_{i=1}^{KN_t}$ is a feasible solution to problem (9). With noting the forms of optimal value functions for the two problems, the optimal value of problem (2) is not greater than that of problem (9). Conversely, if $\{\underline{s}_i^*\}_{i=1}^{KN_t}$ is given as the optimal solution to problem (2) with $E_{(G,\text{total})} = 0$, *i.e.*, it satisfies the optimization problem next:

$$\begin{aligned} \max_{\{s_i\}_{i=1}^{KN_t}} \quad & \sum_{i=1}^{KN_t} w_i \log(1 + a_i s_i) \\ \text{subject to: } & 0 \le s_i, \ \forall i; \\ & \sum_{i \in \chi_k} s_i \le \overline{P}_k, \ 1 \le k \le K; \\ & \sum_{k=1}^{l} \sum_{i \in \chi_k} s_i \le \sum_{k=1}^{l} E_{in}(k), \\ & 1 \le l \le K, \end{aligned}$$

let $\{s_i^*\}_{i=1}^{K}$ be the optimal solution to the implied problem (9). Hence, $s_i^* - \underline{s}_i^* \ge 0, \forall i$, stemming from Lemma 3. It implies, with Lemmas 3-4, that $\{s_{H,i}, s_{G,i}\}_{i=1}^{KN_t}$ is a feasible solution of problem (2), as $s_{H,i}$ is assigned by $\underline{s}_i^*$ and $s_{G,i}$ is assigned by $s_i^* - \underline{s}_i^*$. The objective function value of problem (2) is equal to that of (9), with respect to the aforementioned $\{s_{H,i}, s_{G,i}\}$ and $\{s_i^*\}$. Then, the former value corresponding to a feasible point is equal to the latter value corresponding to the optimal point. It leads to the maximum objective function value of problem (2) is not less than that of (9), together with the previously proven fact that the maximum objective function value of problem (2) is not greater than that of (9). Therefore, the maximum objective function value of problem (2) is equal to that of (9).

Second, to compute an optimal solution to problem (2), we need to compute the $\{s_i^*\}_{i=1}^{KN_t}$, as the optimal solution to problem (9) and the $\{\underline{s}_i^*\}_{i=1}^{KN_t}$, as the optimal solution to problem (2) with $E_{(G,\text{total})} = 0$. Further, $s_{H,i}$ is assigned by $\underline{s}_i^*$ and $s_{G,i}$ is assigned by $s_i^* - \underline{s}_i^*, \forall i$. Therefore, the obtained $\{s_{H,i}, s_{G,i}\}$ is an optimal solution to problem (2). Thus, GWFGUP is utilized $K+1$ times to obtain $\{s_{H,i}, s_{G,i}\}$ correspondingly. In addition, $\{\underline{s}_i^*\}_{i=1}^{KN_t}$, as the optimal solution to problem (2) with $E_{(G,\text{total})} = 0$, implies that PAMEC(K) can utilize the harvested energy, most efficiently, as the first priority for the (power) allocation. Since the two sets of optimal solutions can all be computed by PAMEC(K), with a finite amount of computation, Proposition 1 is thus proven. □

IV. Numerical Results

This section consists of two parts: numerical examples and computational complexity analysis. The first part uses a few examples to account for the procedure of PAMEC(K), to compare with PD-IPM, and to illustrate the performance of the PAMEC(K) with the number of the antennas increasing upto a massive MIMO case. As a side note, PD-IPM cannot be used for a large-scale optimization problem [26, p. 616]. The second part discusses computational complexity of the proposed algorithm, and arrives at the conclusion of polynomial complexity with low degree. Due to exploiting the structure of the proposed problems, the proposed algorithms show significant efficiency, indeed. For clearly understanding the procedure of PAMEC, a simple example is first introduced.

Example 1: Assume that there are three epochs, each with the same length ($L_k = 2, k = 1, 2, 3$) and the unit weight ($w_k = 1, k = 1, 2, 3$). Let $N_t = 2$. Suppose the fading profile for the three epochs is

$$\mathbf{H}_1 = \frac{1}{\sqrt{12}}\begin{pmatrix} 1 & -1 \\ \sqrt{2} & \sqrt{2} \end{pmatrix}, \quad \mathbf{H}_2 = \frac{1}{\sqrt{12}}\begin{pmatrix} \sqrt{3} & -\sqrt{3} \\ 2 & 2 \end{pmatrix} \text{ and}$$
$$\mathbf{H}_3 = \frac{1}{\sqrt{12}}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ \sqrt{6} & \sqrt{6} \end{pmatrix}.$$

At the beginning of each epoch, the energy is harvested ($E_{(G,\text{total})} = 1$; $E_{in}(1) = 12, E_{in}(k) = 2, k = 2, 3$). Also, the group upper bound constraints of the powers are $\overline{P}_k = 2 + 6|(k-2)| = \{8, 2, 8\}, k = 1, 2, 3$.

From given $\mathbf{H_1}$, $\frac{1}{a_1} = 6$ and $\frac{1}{a_2} = 3$ are illustrated as the the heights of the first two stairs in Fig. 3(a). Similarly, we can have heights for the remaining four steps as 2, 1.5, 1.2, and 1 respectively. Carrying out GWFGUP(3,3), the power allocation to epoch 3 is illustrated in Fig. 3(b), and then the heights to represent the power allocation by GWFGUP(3,3) plus the heights of the original steps are formed into heights of the new steps. Carrying out GWFGUP(2,3), the power allocation to

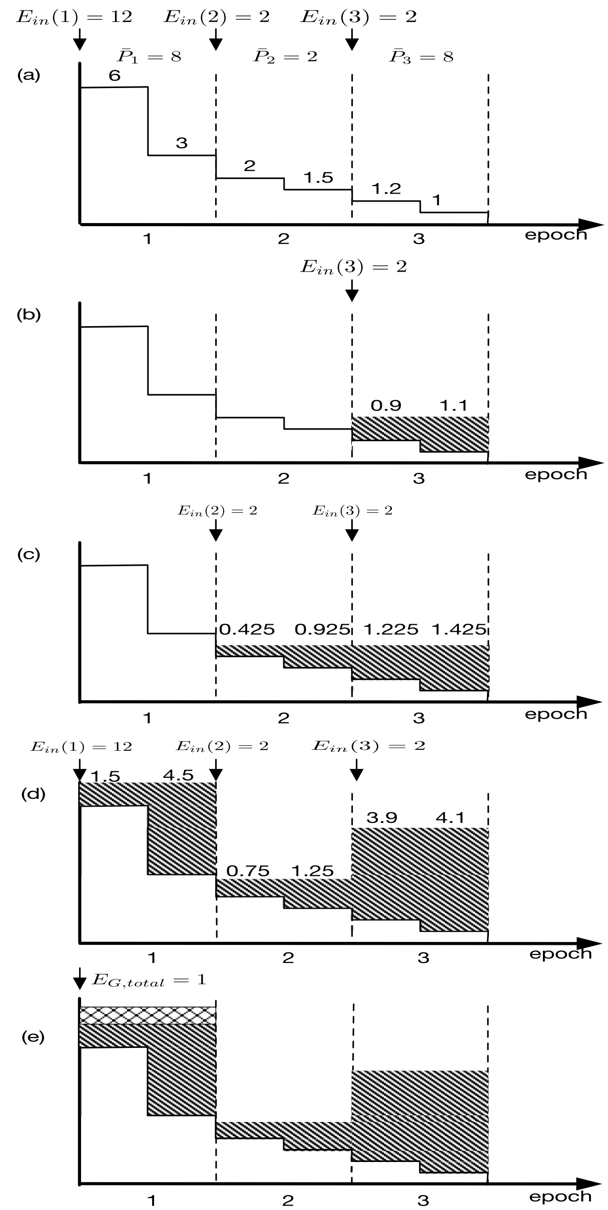

Fig. 3. Illustration for Algorithm PAMEC(K) ($K = 3$, $N_t = 2$).

epochs 2-3 is illustrated in Fig. 3(c), and then similarly updating heights of the newer steps. Carrying out GWFGUP(1,3), the power allocation to epochs 1-3 is illustrated in Fig. 3(d), and then similarly further updating heights of the newest steps. Finally, Fig. 3(e) carries out GWFGUP(1,3) with "filling the amount of water:" $E_{(G,\text{total})}$ into epochs 1-3 as shown in the grid area in Fig. 3(e). Then the optimal power allocation to problem (2) is obtained.

Example 1 is calculated out by applying GWFGUP, satisfying the mixed constraints of (1), from epoch 3, to epochs 2-3 and up to epochs 1-3. At the final step, the completed optimal solution is obtained. Fig. 3(d) shows that the water level non-decreasing condition, as the optimal solution condition for energy harvesting transmission with $E_{max} \gg 0$ (refer to [17], [20]), does not hold. For a summary of the algorithm PAMEC(K): PAMEC(K) can compute the optimal solution of the target problem in finite steps. It does not need to

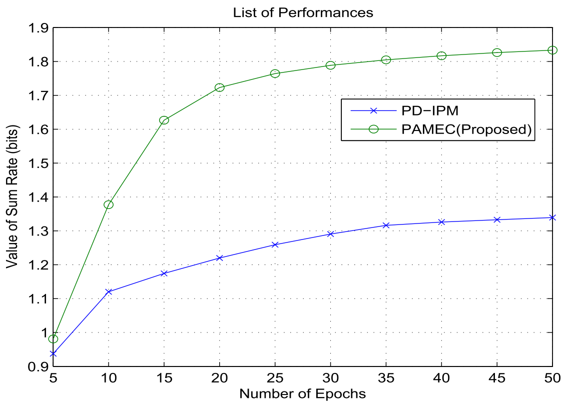

Fig. 4. Comparison of Achieved Throughput in Epochs.

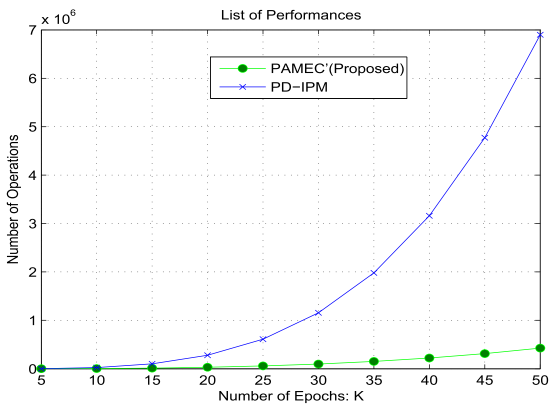

Fig. 5. Comparison of Computational Complexity (Number of Operations).

solve any non-linear system, consisting of many equations and inequalities in the multiple original variables and dual ones.

Example 2: The well known optimization algorithm over the real space, the primal-dual interior point method (PD-IPM), is chosen for comparison purpose due to its competitiveness in computing the solutions to the convex optimization problems. Note that PD-IPM cannot directly be used for (1) in several complex optimization variables. Thus we may lent our proposed real model of (2) to PD-IPM. Without loss of generality, assume $N_t = 2$ too.

Fig. 4 and Fig. 5 are used to show the difference between PD-IPM and PAMEC(K) for the maximum throughput problems as a function of the number of epochs. Channel gains are generated randomly using random variables with each entry drawn independently from the standard Gaussian distribution. $\{E_{(G,\text{total})}, \{E_{in}(k)\}\}$ are the set of randomly chosen positive numbers. The epoch power upper bound constraints in (1) are taken as $\{3k\}_{k=1}^{K}$. A group of different weights are also generated randomly. The chosen parameters mentioned above are assigned to both algorithms with the identical values for fair comparison.

In these two figures, the circle markers and the cross markers represent the results of the proposed PAMEC(K) and PD-IPM respectively. Fig. 4 compares the achieved throughput of two algorithms. For the proposed PAMEC(K), since it uses recursion, no iteration is invoked. For PD-IPM, the obtained throughput is the result after 120 iterations.

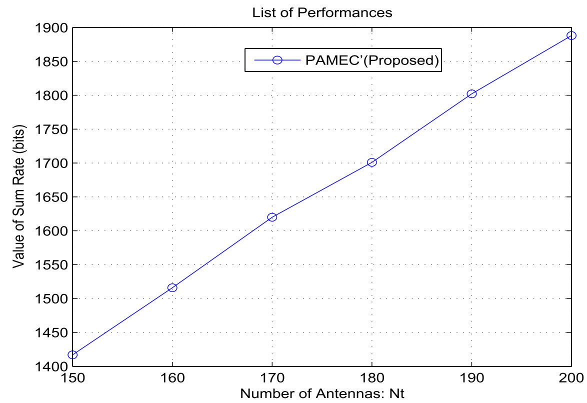

Fig. 6. Power Allocation of PAMEC as K=10 and the N_ts.

Significant improvement in throughput can be achieved especially when K is greater than 20 epoches. Fig. 5 compares the number of operations to achieve the throughput presented in Fig. 4. With the increasing of the number of epoches, the computation amount of the proposed PAMEC increases almost linearly with a slow slope. However, the computation amount of PD-IPM increases exponentially. In the most discrepancy point, when $K = 50$, the number of operation of the PAMEC is about 0.42 million fundamental logical and arithmetic operations, while PD-IPM uses about 6.9 million fundamental logical and arithmetic operations. These results show that the proposed PAMEC exhibits significant performance enhancement and efficiency in computation.

Example 3: Since (1) and (2) are equivalent, we may use our proposed real model of (2). This example uses large N_ts. Without loss of generality, assume $N_t = \{150, 160, \ldots, 190, 200\}$ and $K = 10$. Often, this number K of epochs is not large, for an acceptable prediction/estimation channel state information. There are KN_t optimization variables or channels for (2). Since PD-IPM cannot guarantee to handle a large-scale optimization [26, p. 616], this example does not use PD-IPM.

Here, Fig. 6 is used to show PAMEC(K) for the maximum throughput problems vs. number of antennas. Channel gains are generated randomly using random variables with each entry drawn independently with the mean of 0.3 and the square variance of 0.1. $\{E_{(G,\text{total})}, \{E_{in}(k)\}\}$ are the set of randomly chosen positive numbers with the mean of 29 dBW. The epoch power upper bound constraints in (2) are taken as those with that of 30 dBW. The equal weights are also taken for clear observation. It can be observed that the throughput improves almost linearly as the increasing of the number of antennas in a large scale massive antenna array.

A. Computational Complexity Analysis

To compute the optimal solution, PAMEC(K) only utilizes GWFGUP $K + 1$ times, so it needs $\sum_{k=1}^{K+1} O(N_t^2 k^2) = O(N_t^2 K^3)$, the degree 5 polynomial complexity, fundamental operations (refer to [19]). In fact, it is seen that the computational complexity of PAMEC(K) is: $\sum_{k=1}^{K}\sum_{m=1}^{k}(8N_t + 3)m + K(K+1)N_t = \frac{1}{6}(8N_t+3)K(K+1)(K+2) + K(K+1)N_t$, as the number of the operations, at the worst case.

It is more accurate than the big O notation of $O(N_t^2 K^3)$. However, PD-IPM needs a polynomial computational complexity: $O(N_t^{3.5} K^{3.5}) \log(1/\epsilon)$, *i.e.*, the degree 7 polynomial complexity, to compute an ϵ solution, but the ϵ solution is not an optimal solution (refer to [26], [33]). It cannot offer a computational complexity with a concrete number of the operations [34], unlike ours. Our method eliminates any linear search but output the exact optimal solution with a finite amount of computation. As a side note, the complexity of $O(N_t^2 K^3)$, as K is fixed, is only a quadratic polynomial in N_t. This point is suitable for exactly solving a massive MIMO throughput maximization problem. Often, K cannot take a large value due to the limitation of precision of prediction for the system parameters.

Generally speaking, PAMEC(K) needs a total of $O(N_t^2 K^3)$ basic operations to compute *the exact (optimal) solution*, while PD-IPM needs a total of $O(N_t^{3.5} K^{3.5}) \log(1/\epsilon)$ basic operations to compute *an ϵ solution*. The algorithm PAMEC(K) has the same level of computational complexity for the target problem, using whether its real form or complex form.

V. Conclusion

For the throughput maximization problem of the PG, MIMO and EH coexisting system with the mixed constraints, we proposed a novel efficient water-filling algorithm (PAMEC) to compute the exact solution. PAMEC(K) stems from GWFGUP, which is used as a functional block of PAMEC. The computation of the proposed algorithm only needs finite steps with a low degree polynomial computational complexity. Numerical examples are provided for illustrating the steps for the exact solution by the proposed algorithm. They indeed show that, for the target problem, our algorithm uses less amount of computation and achieves the optimal system throughput, especially for large scale systems, *e.g.* the massive MIMO system, while the existing optimization methods cannot guarantee to compute the exact solution, even including the most efficient primal-dual interior point method.

Appendix A
Proof of Lemma 2

To clearly understand, before the formal proof, two facts need reminding. The first fact is that the new problem that changes the objective function of (2) into $\sum_{i=1}^{N_t \times K} w_i \log(\frac{1}{a_i} + s_i)$ with respect to keeping the original constraints of (2), is equivalent to (2). This equivalence is easily obtained, from the proposed Definition 2.3. It implies that the equivalent optimization problems have the same set of optimal solutions. Based on this meaning of the same set of optimal solutions, equivalent optimization problems should be identical. Since Line 5 in the statement of PAMEC(K) continuously updates the channel gains, the second fact is that the reciprocal of the current ith channel gain is equal to that of the original ith channel gain, $\forall i$, plus the sum of the ith entries of the solutions that have continuously been obtained by GWFGUP at Line 3 in the statement of PAMEC(K). To distinguish between both of the channel gains, let us denote the current and the original channel gains by $\{a_i\}$ and $\{a_i^{(0)}\}$ respectively. In addition, any epoch in the process and its following epochs can form a new process. Thus, for this new process, it is seen that the target problems (1) and (2) only need a few of changes on the subscripts. For example, (1) changes the lower bound of the summation of its objective function from 1 to the index of the chosen initial epoch, and so on. This new process is used for the following mathematical induction.

Mathematical induction is carried out with respect to the index L. It corresponds to Lines 2-9 in the statement of PAMEC(K). As $L = K$, *i.e.* the process only containing epoch K, the conclusion holds naturally. This is because only Lines 3, 4 and 7 of PAMEC(K) are used. As $1 < L \leq K$, *i.e.* the process only containing epochs from L upto K, assume that $\{s_{H,i}^*\}_{i \in \cup_{L \leq k \leq K} \chi_k}$ is the solution to (2) with the initial epoch L. Through the two facts, it is seen that any member or entry $s_{H,i}^*$ of the solution, mentioned above, plus the reciprocal of $a_i^{(0)}$, is the the reciprocal of the current channel gain $a_i, \forall i \in \chi_k, L-1 \leq k \leq K$, when L at Line 2 of PAMEC(K) is just regressed into $L-1$. Here, the optimization problem in the increment variables $\{\triangle s_{H,i}\}$, corresponding to GWFGUP($L-1$, K, $E_{in}(L-1)$, $\{\overline{P}_k\}_{k=L-1}^K$), is:

$$
\begin{aligned}
\max_{\{\triangle s_{H,i}\}_{i \in \cup_{k=L-1}^K \chi_k}} \quad & \sum_{k=L-1}^{K} w_k \\
& \sum_{i \in \chi_k} \log\Big(\frac{1}{a_i^{(0)}} + s_{H,i}^* + \triangle s_{H,i}\Big) \\
\text{subject to: } & 0 \leq \triangle s_{H,i}, \;\; \forall i \in \cup_{k=L-1}^K \chi_k; \\
& \sum_{i \in \chi_k} \triangle s_{H,i} \leq \overline{P}_k - \sum_{i \in \chi_k} s_i^*, \\
& L-1 \leq k \leq K; \\
& \sum_{k=L-1}^{K} \sum_{i \in \chi_k} \triangle s_{H,i} \\
& \leq E_{in}(L-1).
\end{aligned} \tag{10}
$$

For the optimal solution $\{\triangle s_{H,i}\}_{i \in \cup_{L-1 \leq k \leq K} \chi_k}$ to (10), there are dual variables $\{\{\mu_{k,i} | i \in \chi_k, L-1 \leq k \leq K\}, \{\sigma_k\}_{k=L-1}^K, \lambda\}$, where three sets of the dual variables correspond to the three sets of constraints of (10) respectively, such that the following KKT conditions of (10) hold,

$$
\begin{cases}
\frac{1}{\frac{1}{a_i^{(0)} w_i} + \frac{s_{H,i}^* + \triangle s_{H,i}}{w_i}} = \lambda + \sigma_k - \mu_{k,i}, \\
\quad \forall i \in \chi_k, L-1 \leq k \leq K; \\
\triangle s_{H,i} \geq 0, \, \mu_{k,i} \triangle s_{H,i} = 0, \, \mu_{k,i} \geq 0, \\
\quad \forall i \in \chi_k, L-1 \leq k \leq K; \\
\sum_{i \in \chi_k} \triangle s_{H,i} \leq \overline{P}_k - \sum_{i \in \chi_k} s_{H,i}^*, \\
\quad \sigma_k [\sum_{i \in \chi_k} (s_{H,i}^* + \triangle s_{H,i}) - \overline{P}_k] = 0, \\
\quad \sigma_k \geq 0, \forall k; \\
\sum_{k=L-1}^{K} \sum_{i \in \chi_k} \triangle s_{H,i} \leq E_{in}(L-1), \\
\quad \lambda [\sum_{k=L-1}^{K} \sum_{i \in \chi_k} \triangle s_{H,i} - E_{in}(L-1)] = 0, \\
\quad \lambda \geq 0.
\end{cases} \tag{11}
$$

If $\sum_{k=L-1}^{K} \sum_{i \in \chi_k} \triangle s_{H,i} < E_{in}(L-1)$, then $\lambda = 0$. It implies that $\sigma_k > 0, \forall k$. Thus, $\sum_{i \in \chi_k} (s_{H,i}^* + \triangle s_{H,i}) = \overline{P}_k, \forall k$. It is seen that such a set of $\{s^*_{H,i} + \triangle s_{H,i}\}$ is an optimal solution to (2), noting that the initial epoch is the aforementioned $L-1$.

On the other hand, assume $\sum_{k=L-1}^{K}\sum_{i\in\chi_k}\triangle s_{H,i} = E_{in}(L-1)$. Let $E_g = \{k|\sum_{i\in\chi_k}\triangle s_{H,i} > 0, L-1 \le k \le K\}$, $\overline{k}_g = \max\{k|k\in E_g\}$ and $\underline{k}_g = \min\{k|k\in E_g\}$, $\Lambda_1 = \{k_1|\lambda < \sum_{k=k_1}^{K}\lambda_k, \overline{k}_g < k_1 \le K\}$ and $\Lambda_0 = \{k_0|\lambda > \sum_{k=k_0}^{K}\lambda_k, L-1 \le k_0 < \underline{k}_g\}$, and $\overline{k}_1 = \max\{k_1|k_1\in\Lambda_1\}$ and $\underline{k}_0 = \min\{k_0|k_0\in\Lambda_0\}$. If $\overline{k}_1 < k \le K$, where $\overline{k}_1 < K$, let $\tilde{\lambda}_k = \lambda_k, \tilde{\nu}_k = \nu_k$ and $\tilde{\mu}_i = \mu_i, \forall i\in\chi_k$. Note that $\{\{\mu_i\}_{i\in\cup_{k=L}^{K}\chi_k}, \{\lambda_k,\nu_k\}_{k=L}^{K}\}$ are the optimal dual variables of (2) with $E_{(G,\text{total})} = 0$, the initial epoch L and the optimal solution $\{s^*_{H,i}\}_{i\in\cup_{L\le k\le K}\chi_k}$. Further, these three sets of $\{\mu_i\}, \{\nu_k\}$ and $\{\lambda_k\}$ correspond to the preceding three sets of constraints for (2) with $E_{(G,\text{total})} = 0$, respectively. If $k = \overline{k}_1$, let $\tilde{\lambda}_k = \lambda - \sum_{m=k+1}^{K}\lambda_m, \tilde{\nu}_k = (\sum_{m=k}^{K}\lambda_m - \lambda) + \nu_k$ and $\tilde{\mu}_i = \mu_i, \forall i\in\chi_k$. Note that the special summation of $\sum_{m=K+1}^{K}\lambda_m$ is defined as zero. If $\overline{k}_g < k < \overline{k}_1$, let $\tilde{\lambda}_k = 0, \tilde{\nu}_k = \nu_k + (\sum_{m=k}^{K}\lambda_m - \lambda)$ and $\tilde{\mu}_i = \mu_i, \forall i\in\chi_k$. If $\underline{k}_g \le k \le \overline{k}_g$, let $\tilde{\lambda}_k = 0, \tilde{\nu}_k = \sigma_k$ and $\tilde{\mu}_i = \mu_{k,i}, \forall i\in\chi_k$. Note that it is seen that, if $\underline{k}_g \le k \le \overline{k}_g$, there does not exist the case of $\sum_{i\in\chi_k}\triangle s_{H,i} > 0$ and $\sum_{m=\max\{k,L\}}^{K}\lambda_m < \lambda$ holding. Also, note that $\{\lambda, \overline{\sigma}_k, \underline{\sigma}_k\}$ are the aforementioned optimal dual variables in the KKT conditions of (11). If $\underline{k}_0 \le k < \underline{k}_g$ and $k \ge L-1$, it is seen that $\sum_{i\in\chi_k}s^*_{H,k} = 0$. Thus, if $\underline{k}_0 \le k < \underline{k}_g$ and $k > L-1$, let $\tilde{\lambda}_k = 0, \tilde{\nu}_k = \nu_k$ and $\tilde{\mu}_i = (\lambda - \sum_{m=k}^{K}\lambda_m) + \mu_i, \forall i\in\chi_k$. If $\underline{k}_0 \le k < \underline{k}_g$ and $k = L-1$, let $\tilde{\lambda}_k = 0, \tilde{\nu}_k = \sigma_k$ and $\tilde{\mu}_i = \lambda + \mu_{k,i}, \forall i\in\chi_k$. If $L-1 \le k < \underline{k}_0$, let $\tilde{\lambda}_k = 0, \tilde{\nu}_k = \sigma_k$ and $\tilde{\mu}_{k,i} = \mu_{k,i}, \forall i\in\chi_k$. Note that the aforementioned cases have been enumerated. For the optimization problem (2) with $E_{(G,\text{total})} = 0$ and the initial epoch $L-1$, $\{s^*_{H,i} + \triangle s_{H,i}\}_{i\in\cup_{k=L-1}^{K}\chi_k}$, as optimization variables, and $\{\tilde{\lambda}_k, \tilde{\nu}_k, \{\tilde{\mu}_i\}_{i\in\chi_k}\}_{k=L-1}^{K}$, as the corresponding dual variables, satisfy the KKT conditions of (2) with $E_{(G,\text{total})} = 0$ and the initial epoch $L-1$. The $\{\tilde{\lambda}_k, \tilde{\nu}_k, \tilde{\mu}_{i\in\chi_k}\}, \forall k$, correspond to the kth causal sum power, group upper bound power and non-negative power constraints of (2) with $E_{(G,\text{total})} = 0$ and the initial epoch $L-1$, respectively. At the same time, the General Constraint Qualification (refer to (3.71) of [35, Th. 3.8] of problem (2) holds. Together with the uniqueness of the optimal solution, which stems from the optimization problem (2), with $E_{(G,\text{total})} = 0$, being convex with a strictly concave objective function, $\{\overline{s}^*_{H,i} = s^*_{H,i} + \triangle s_{H,i}\}_{i\in\cup_{k=L-1}^{K}\chi_k}$ is the optimal solution to (2), with $E_{(G,\text{total})} = 0$ and the initial epoch $L-1$.

Therefore, according to the mathematical induction, the conclusion of Lemma 2 is indeed true.

Appendix B
Proof of Lemma 3

Similar to the proof of Lemma 2, the proof of Lemma 3 can also be obtained. This proof only changes the solution $\{s^*_{H,i}\}_{i\in\cup_{L\le k\le K}\chi_k}$ in the assumption of the mathematical induction, by Lemma 2, into the solution to (2) with $E_{(G,\text{total})} = 0$ that is $\{s^*_{H,i}\}_{i\in\cup_{1\le k\le K}\chi_k}$; then $\triangle s_{H,i}(\ge 0)$ into $\triangle s_i, \forall i$; and obtaining $\{\overline{s}^*_i = s^*_{H,i} + \triangle s_i\}_{i=1}^{KN_t}$. The detail of this proof can copy the large part of Lemma 2 with the aforementioned changes, starting from the assumption of the mathematical induction by Lemma 2. If the changes are noted, we can easily write down this proof. Thus, the outline of this proof has been stated.

Therefore, the conclusion of Lemma 3 is indeed true.

Appendix C
Proof of Lemma 4

According to Lemmas 2-3 and their proofs, $\overline{s}^*_i = s^*_{H,i} + \triangle s_{G,k}, \forall i$, where $\{\triangle s_{G,i}\}_{i=1}^{KN_t}$ is the optimal solution to the following optimization problem:

$$\begin{aligned}\max_{\{\triangle s_i\}_{i\in\cup_{k=1}^{K}\chi_k}} \quad & \sum_{k=1}^{K} w_k \sum_{i\in\chi_k}\log\left(\frac{1}{a_i^{(0)}} + s^*_{H,i} + \triangle s_i\right)\\ \text{subject to:} \quad & 0 \le \triangle s_i,\ \forall i;\\ & \sum_{i\in\chi_k}\triangle s_i \le \overline{P}_k - \sum_{i\in\chi_k}s^*_{H,i},\\ & 1\le k\le K;\\ & \sum_{i=1}^{KN_t}\triangle s_i \le E_{(G,\text{total})}.\end{aligned} \tag{12}$$

Thus, $\sum_{i=1}^{KN_t}\triangle s_{G,i} \le E_{(G,\text{total})}$. Then, $\sum_{i=1}^{KN_t}(\overline{s}^*_i - s^*_{H,i}) \le E_{(G,\text{total})}$, from $\{\triangle s_{G,i} = \overline{s}^*_i - s^*_{H,i}\}$.

Therefore, the conclusion of Lemma 4 is true.

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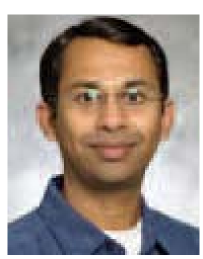

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