

Performance of Coding-Spreading Tradeoff in DS-CDMA Systems Using RCPT and RCPC Codes

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Abstract—The use of rate-compatible punctured turbo and rate-compatible punctured convolutional (RCPT/RCPC) codes as channel codes in a direct-sequence code-division multiple-access system where the system bandwidth expansion is fixed is investigated. The best RCPC and RCPT code rate in terms of maximizing the system spectral efficiency and minimizing the optimal power allocation where the receiver is either a matched filter (MF) or a minimum mean-square error (MMSE) device is assessed. It is shown that for the MF receiver, the coding-spreading tradeoff favors a code-rate reduction. In the case of the MMSE receiver, when the E_b/N_0 value and the system load are increased, the best code rate also increases. By examining the slope of the performance curves, it is deduced that, under similar operating conditions, the best code rate of the RCPT codes is lower than that of the RCPC codes. Also, the best code rate for a Rayleigh fading channel is lower than that for an additive white Gaussian noise channel.

Index Terms—Code-division multiple access (CDMA), coding-spreading tradeoff, linear receivers, rate-compatible punctured codes, rate-compatible punctured turbo and rate-compatible punctured convolutional (RCPT/RCPC) codes.

I. INTRODUCTION

THE rate-compatible punctured convolutional (RCPC) codes proposed by Hagenauer [1] provide a flexible way to implement variable-rate codes with the same encoder and decoder structures. This puncturing philosophy can be applied to turbo codes to generate a family of rate-compatible punctured turbo (RCPT) codes [2], [3]. In this letter, we investigate system performance of RCPC/RCPT-coded direct-sequence code-division multiple-access (DS-CDMA) systems under the constraint that the system bandwidth expansion is fixed. The performance gain in such spreading and coding systems is twofold: the spreading gain offered by DS spreading techniques; and the coding gain offered by channel-coding techniques. Consequently, there exists a tradeoff between the error-correction capability of channel coding and the interference-suppression capability of DS spreading for a fixed-bandwidth expansion scheme.

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In recent years, there has been a large body of work on coding-spreading tradeoff reported in the literature. Most of the reported results on coding-spreading tradeoff pertain mainly to spectral efficiency maximization using random spreading functions and ideal coding for arbitrarily reliable transmission [4]–[7]. The best code rate depends on the receiver structure. For a matched-filter (MF) receiver, spectral efficiency is maximized by letting the whole bandwidth expansion due to channel coding. However, this may not be the case in noncoherent demodulation models [8], in some specific code family [9], or in a jamming environment [10]. For a minimum mean-square error (MMSE) receiver, there exists a best bandwidth-expansion allocation between coding and spreading for sufficiently high E_b/N_0 values. With the MMSE receiver, coding-spreading tradeoff analysis has also been reported in terms of bit-error rate (BER) and cutoff rate [11], achievable capacity [12], spectral efficiency by using convolutional codes in additive white Gaussian noise (AWGN) channels [13], and RCPC codes [14] in AWGN and Rayleigh fading channels.

In this letter, we investigate the coding-spreading tradeoff problem by employing RCPC and RCPT codes as variable-rate channel codes under a large system assumption [15]. System spectral efficiency and optimal power allocation are used as system performance measures. The work reported here aims at providing design guidelines for systems using RCPC/RCPT codes as channel codes, subject to a target BER specification.

II. SYSTEM DESCRIPTION AND APPLIED CHANNEL CODES

We consider a coded synchronous DS-CDMA system where the receiver is either a conventional MF or an MMSE device. The channel is modeled as either an AWGN channel or a Rayleigh fading channel. It is assumed that the spreading sequences are randomly and independently chosen, and that the spreading factor is large enough to satisfy the assumption that the multiple-access interference (MAI) can be approximated as AWGN. Since for a fixed-bandwidth expansion system, the code rate R uniquely determines the spreading factor, we only need to determine the best code rate. For convenience, in what follows, the spreading factor is treated as a real instead of an integer number.

The RCPC encoder consists of an ordinary convolutional encoder with parent-code rate $1/4$ and constraint length 9. The generator polynomials for the parent code are [473, 513, 671, 765] in octal notation. Rates higher than the parent-code rate are generated by puncturing with period 8, and lower rates by nesting. The puncture and nesting patterns used are based on those reported in [16]. The range of RCPC code rates is

$$R_c = \left[\frac{1}{10}, \frac{1}{9}, \dots, \frac{1}{5}, \frac{1}{4}, \frac{8}{31}, \frac{8}{30}, \frac{8}{29}, \dots, \frac{8}{9} \right]. \quad (1)$$

The required signal-to-interference ratio (SIR) for each information bit to achieve a target BER is obtained by using error bounds [1], [16] for both AWGN and Rayleigh fading channels.

The turbo encoder under study consists of two identical recursive systematic convolutional (RSC) encoders in parallel, separated by a pseudorandom interleaver. The RSC encoder has feedforward/feedback polynomials (15/13, 17/13) in octal representation, with constraint length 4 and leading parent-code rate 1/5. The parity-bit streams are punctured to generate a higher rate code in the RCPT code family. The puncturing patterns for the basic-rate code rates 1/4, 1/3, and 1/2 are based on the cdma2000 proposal [17]. It is indicated in [18] that for most of the code rates, when a pseudorandom interleaver is used, the selection of puncturing pattern does not have a significant effect on the code performance. Therefore, for other code rates, we simply choose the puncturing patterns and puncture the two parity streams evenly, without considering further implications on the coding performance. Furthermore, the interleaver length is 1024 information bits, using random interleaving. The puncturing period used is 8, identical to that for the RCPC codes. The resultant code rates are

$$R_t = \left[\frac{1}{5} \quad \frac{1}{4} \quad \frac{8}{31} \quad \frac{8}{30} \quad \frac{8}{29} \quad \cdots \quad \frac{8}{10} \right]. \quad (2)$$

An iterative, soft-input/soft-output (SISO) decoding algorithm is employed for turbo decoding. The decoded BER is obtained after five iterations. This is based on the conclusion drawn in [19] that the gain in error-rate performance saturates at about ten iterations, and that the incremental gain beyond five iterations is not very significant.

III. PERFORMANCE MEASURES

In a coded CDMA system, both coding and spreading result in bandwidth expansion. Using a lower rate channel code will result in a decrease in the spreading factor, which leads to a reduction in the interference-suppression capability. On the other hand, the direct advantage of using lower rate codes is that a lower SIR requirement can be achieved to satisfy the target BER. The factor that contributes to a reduction in SIR requirement is the *coding gain*. The expectation is that the coding gain will allow mobiles to operate at an SIR level that is low enough to offset the negative effect of the decrease in the spreading factor. In the following, system spectral efficiency and optimal received power will be used as system performance measures to specify the best code rate.

A. Spectral Efficiency

We consider the situation in which all the users admitted into the system have the same BER requirement and the same data rate. The system spectral efficiency η is defined as the ratio of the average number of information bits transmitted per unit time to the total bandwidth consumption. Let K , R_b , and W represent the total number of active users, required data rate, and channel bandwidth, respectively. The spectral efficiency is then $\eta = KR_b/W = K/G$, where $G = W/R_b$ is the total processing gain resulting from spreading and coding.

The coded symbol signal-to-noise ratio (SNR) for each user is given by RE_b/N_0 , where R is the applied code rate for either

RCPC or RCPT codes. Let β be the symbol SIR at the output of the receive filter. In an AWGN channel, for a large system, the asymptotic output SIR as a function of the basic system parameters converges with probability to [6], [15]

$$\beta = \begin{cases} \frac{RE_b}{N_0 + \eta E_b} & \text{(MF)} \\ \frac{RE_b}{N_0 + \eta \frac{E_b}{1 + \beta}} & \text{(MMSE)}. \end{cases} \quad (3)$$

The spectral efficiency can be readily expressed as

$$\eta = \begin{cases} \left(\frac{R}{\beta} - \frac{1}{E_b} \right) & \text{(MF)} \\ (1 + \beta) \left(\frac{R}{\beta} - \frac{1}{E_b} \right) & \text{(MMSE)}. \end{cases} \quad (4)$$

The bit SIR is $\gamma = \beta/R$ with code rate R . Substituting γ into (4) yields

$$\eta = \begin{cases} \left(\frac{1}{\gamma} - \frac{1}{E_b} \right) & \text{(MF)} \\ \left(\frac{1}{\gamma} + R - \frac{R\gamma}{E_b} - \frac{1}{E_b} \right) & \text{(MMSE)}. \end{cases} \quad (5)$$

From (5), it can be seen that for the MF receiver, for a given E_b/N_0 , the lower the output γ , the higher the spectral efficiency can be achieved. Consequently, a lower code rate is more desirable than a longer spreading, as long as the code rate reduction can result in a lower target SIR requirement.

However, the spectral efficiency for the MMSE receiver is not a monotonic function of the code rate. The first and the third terms inside the brackets on the right-hand side of (5) prefer a lower rate, but the second term inside the brackets prefers a higher rate. When E_b/N_0 is low, the third term contributes more and pushes the best code rate toward the lower rate direction. From (4), it is seen that the gain of the MMSE receiver over the MF receiver is reflected by the term $(1 + \beta)$. A higher gain needs a higher β , which means a lower coding gain. Thus, selecting the best code rate is an important exercise.

For a Rayleigh fading channel, Biglieri *et al.* [20] show that, apart from a scale factor, the SIR for both MF and MMSE receivers has the same probability distribution as the fading power gain. Let z denote the fading channel power gain, and β be the SIR when the average fading power is one. Then the receiver output SIR can be written as $\beta = z\tilde{\beta}$. From [20, eq. 24], $\tilde{\beta}$ can be written in the form

$$\tilde{\beta} = \frac{RE_b}{N_0 + \eta E_b \zeta(\tilde{\beta})} \quad (6)$$

where

$$\zeta(\tilde{\beta}) = \begin{cases} E[z] & \text{(MF)} \\ E\left[\frac{z}{1+z\tilde{\beta}}\right] & \text{(MMSE)}. \end{cases} \quad (7)$$

The spectral efficiency can then be expressed as

$$\eta = \frac{1}{\zeta(\tilde{\beta})} \left[\frac{R}{\tilde{\beta}} - \frac{1}{N_0} \right]. \quad (8)$$

For Rayleigh fading, z follows an exponential distribution. On the assumption that the average fading power is one, $\zeta = 1$ for the MF receiver. For the MMSE receiver

$$\begin{aligned} \zeta(\tilde{\beta}) &= \int_0^\infty \frac{z}{1+z\tilde{\beta}} \cdot e^{-z} dz \\ &= \frac{1}{\tilde{\beta}} \left[1 - \frac{1}{\tilde{\beta}} e^{1/\tilde{\beta}} \int_1^\infty \frac{e^{-t/\tilde{\beta}}}{t} dt \right]. \end{aligned} \quad (9)$$

The spectral efficiency of the MF receiver in a fading channel in (8) has the same formula as that in an AWGN channel in (4). For the MMSE receiver, the factor $(1 + \beta)$ in (4) is replaced by $1/\zeta(\tilde{\beta})$ in (8). From (7), it is observed that $\zeta(\tilde{\beta})$ is a monotonically decreasing function of $\tilde{\beta}$. Therefore, in order to maximize η in (8), the first term prefers a larger $\tilde{\beta}$, while the second term prefers a smaller $\tilde{\beta}$.

B. Optimal Power Allocation

The optimal received power is defined as the received power causing the least interference to other signals while satisfying the corresponding quality of service (QoS) requirement. Let β_T be the required symbol SIR at the output of the Viterbi decoder for the RCPC codes, or the SISO decoder for the RCPT codes, to meet the QoS requirement. Satisfactory operation requires $\beta \geq \beta_T$. When $\beta = \beta_T$, the power allocation is said to be optimal. Let σ^2 be the AWGN noise power at the output of the receiver filter. The receiver filter bandwidth corresponds to the symbol rate R_s , which is given as $R_s = R_b/R$. As a result, the noise power is $\sigma^2 = N_0 R_s$. Based on this definition, the optimal received power in an AWGN channel can be derived as

$$P = \begin{cases} \frac{\beta\sigma^2}{1 - \frac{\eta}{R}\beta} & \text{(MF)} \\ \frac{\beta\sigma^2}{1 - \frac{\eta}{R}\frac{\beta}{1+\beta}} & \text{(MMSE)}. \end{cases} \quad (10)$$

In the form of (10), it is not readily inferable what code rate would lead to the minimum power allocation. Normalizing P by $N_0 R_b$ leads to the required signal-to-noise spectral density ratio $(E_b/N_0)_{\text{req}} = P/N_0 R_b$. Since $N_0 R_b$ is a constant, optimal power allocation is equivalent to minimizing $(E_b/N_0)_{\text{req}}$. Dividing both sides of (10) by $N_0 R_b$ and manipulating, $(E_b/N_0)_{\text{req}}$ can be expressed as

$$\left(\frac{E_b}{N_0} \right)_{\text{req}} = \begin{cases} \frac{\gamma}{1 - \eta\gamma} & \text{(MF)} \\ \frac{\gamma}{1 - \frac{\eta}{\gamma + R}} & \text{(MMSE)}. \end{cases} \quad (11)$$

From (11), it can be seen that given the system load η for the MF receiver, the optimal $(E_b/N_0)_{\text{req}}$ is minimized if and only if γ is minimized. This again means that, as long as the coding gain is nonnegative when further reducing the code rate, the tradeoff favors coding. For the MMSE receiver, the contribution

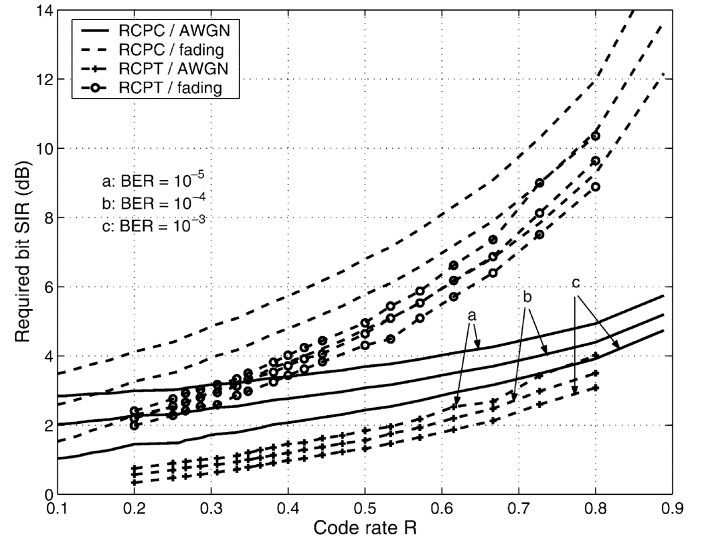


Fig. 1. Required SIR as a function of the code rate by using RCPT and RCPC codes, with target BER = $[10^{-3}, 10^{-4}, 10^{-5}]$.

of the numerator is the same as that of the MF receiver, but the term $(1/\gamma + R)$ in the denominator contributes to the optimal $(E_b/N_0)_{\text{req}}$ level.

Similarly, the optimal power in a Rayleigh fading channel can be derived as

$$P = \frac{\tilde{\beta}\sigma^2}{1 - \frac{\eta}{R}\tilde{\beta}\zeta(\tilde{\beta})} \quad (12)$$

and the corresponding signal-to-noise spectral density ratio $(E_b/N_0)_{\text{req}}$ is

$$\left(\frac{E_b}{N_0} \right)_{\text{req}} = \frac{\gamma}{1 - \frac{\eta}{R}\tilde{\beta}\zeta(\tilde{\beta})} \quad (13)$$

where $\zeta(\tilde{\beta}) = 1$ for an MF receiver and, for the MMSE receiver, it is given by (9).

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical results to illustrate the coding-spreading tradeoff in terms of the spectral efficiency and the optimal power allocation are presented.

Fig. 1 shows the required bit SIR as a function of the code rate for both RCPC and RCPT codes in AWGN and Rayleigh fading channels. The results for RCPC codes are obtained by using the union bounds, while those for the RCPT codes are obtained by simulation. The three curves in each group in Fig. 1 represent the target BER of 10^{-5} to 10^{-3} , from top to bottom. It is observed that the required SIR and the slope of the curves decrease as the code rate decreases. It can also be seen that all three curves in each group exhibit a similar trend. In the results to follow, we use 10^{-5} as the target BER.

Figs. 2 and 3 show the spectral efficiency by using the RCPC and RCPT codes in AWGN and Rayleigh fading for the MF and MMSE receivers, respectively. The spectral efficiency is obtained by using (5) and (8), and the required bit SIR values given in Fig. 1. The values of E_b/N_0 are $[6, 8, 10]$ dB from bottom to top within each group of curves. Fig. 2 shows that the spectral efficiency for the MF receiver monotonically increases as the

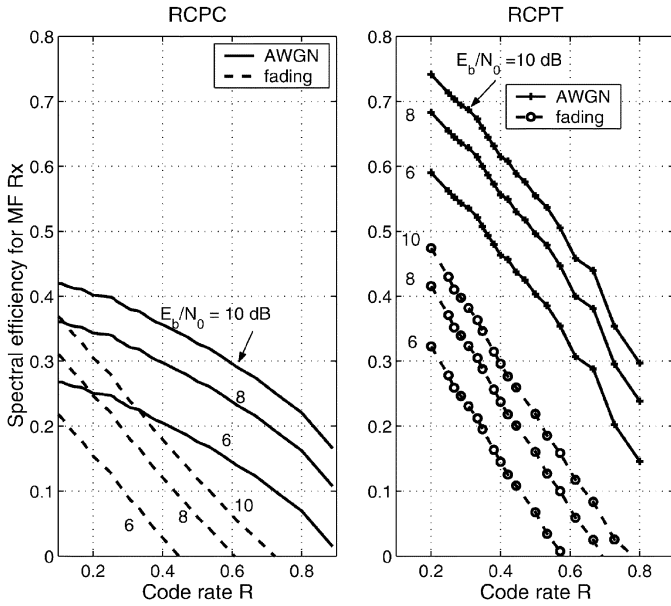


Fig. 2. Spectral efficiency by using RCPC and RCPT codes for MF receiver, with $E_b/N_0 = [6, 8, 10]$ dB.

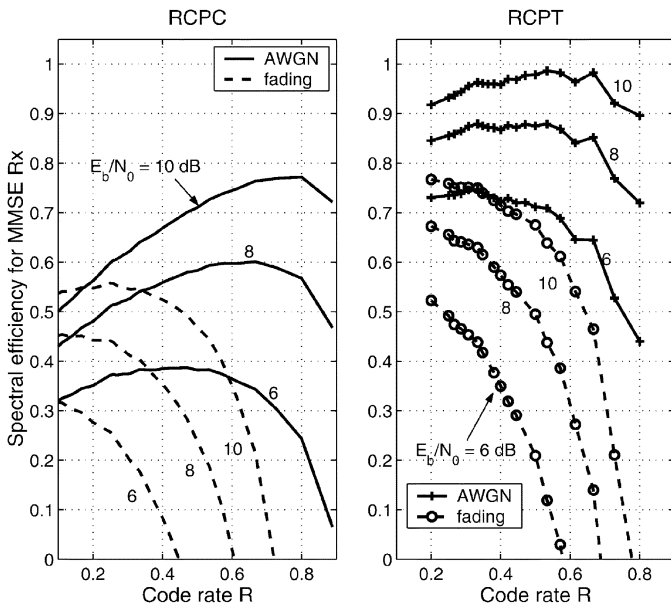


Fig. 3. Spectral efficiency by using RCPC and RCPT codes for MMSE receiver, with $E_b/N_0 = [6, 8, 10]$ dB.

code rate decreases, while Fig. 3 shows that the spectral efficiency performance for the MMSE receiver exhibits concavity when E_b/N_0 is larger than a threshold. For the MF receiver, from (5) and (11), which also apply to fading channels, we can find that the performance depends on the reciprocal of the target SIR. Fig. 1 shows that inside the range of the studied code rates, the target SIR decreases monotonically with the applied code rate. As a result, in the MF case, the best code rate in our code family is the lowest one for both RCPC and RCPT codes. Due to space limitations, in what follows, we present performance results pertaining to the MMSE receiver only.

From the coding-spreading tradeoff point of view, the MMSE receiver offers a richer structure than the MF receiver. Fig. 4 shows the variation of the best code rate as a function of E_b/N_0 . As can be seen from Figs. 3 and 4, when E_b/N_0 increases,

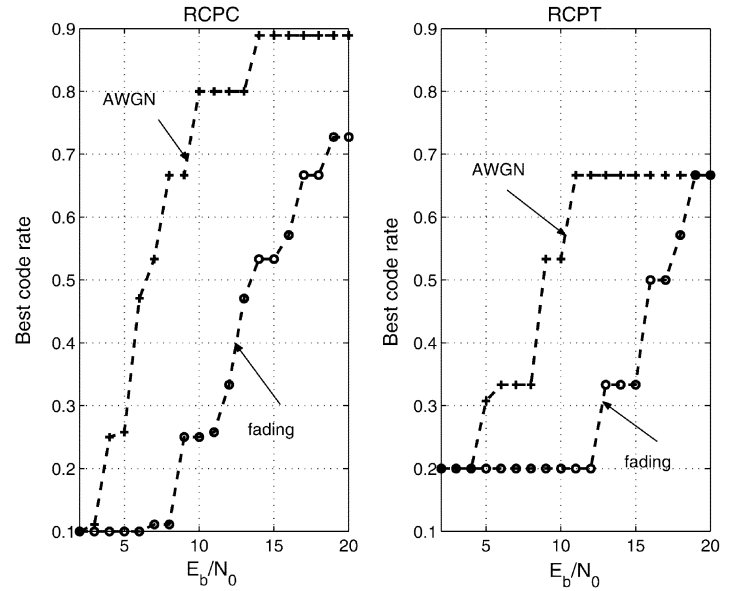


Fig. 4. Best code rate to maximize the spectral efficiency with respect to E_b/N_0 .

the best code rate shifts to the higher rate direction for both the RCPC and RCPT codes. This observation is consistent with (5). With a smaller E_b/N_0 , the third term, $R\gamma/(E_b/N_0)$, in (5) which favors coding, begins to contribute more to the spectral efficiency, and pushes the best code rate to the lower rate direction. In addition, compared with the RCPC codes, the RCPT codes achieve a higher spectral efficiency and a lower best code rate for the same E_b/N_0 value. The best code rate in Rayleigh fading is also lower than that in AWGN. The reason for this discrepancy can be traced back to Fig. 1, where it is shown that the slope of the performance curves in Rayleigh fading is sharper than that in AWGN, and the slope of the performance curves for the RCPT codes is sharper than that for the RCPC codes, for the same channel condition.

Fig. 5 plots the optimal signal-to-noise spectral density ratio, $(E_b/N_0)_{\text{req}}$, as a function of the applied RCPC/RCPT code rates, with system load as a parameter, in both AWGN and Rayleigh fading channels. These results have been obtained using (11) and (13), and the target SIR from Fig. 1. It can be seen that the best code rate depends on the system load. The higher the system load, the larger will be the best code rate. This suggests that, as the load increases, a larger proportion of the bandwidth expansion should be allocated to DS spreading to suppress the increased MAI. On the other hand, for a single-user transmission, no gain is expected by using DS spreading, and all the processing gain should be allocated to error-correction coding. In addition, it can be observed that as the system load increases, the optimal $(E_b/N_0)_{\text{req}}$ increases faster in the lower code-rate region. Thus, caution should be taken in using low-rate codes when designing a high-load system.

Finally, Fig. 6 shows the optimal $(E_b/N_0)_{\text{req}}$ from (11) and (13) as a function of the applied code rate, with target BER as a parameter. It can be seen that the best code rate for the RCPC codes moves faster in the higher code rate direction as the target BER becomes more stringent. For the RCPT codes, the curves within each group appear to differ only by a fixed scaling factor.

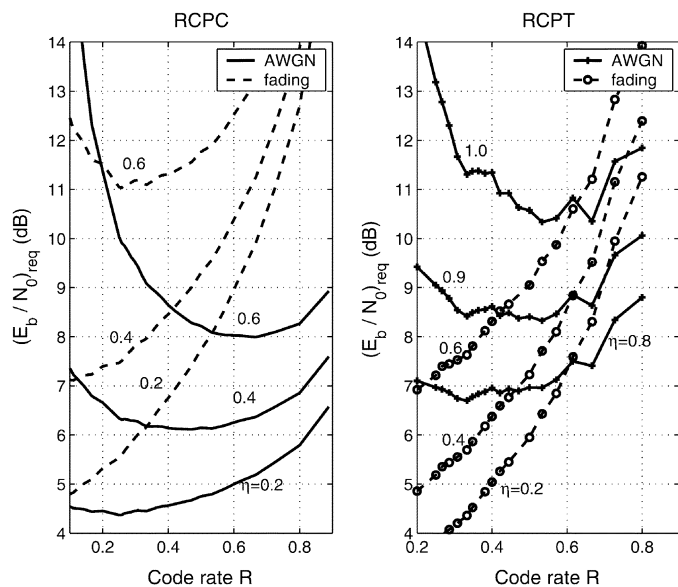


Fig. 5. Optimal signal-to-noise spectral density ratio, $(E_b/N_0)_{req}$, as a function of the code rate and system load. RCPC: $\eta = [0.2, 0.4, 0.6]$. RCPT: $\eta = [0.8, 0.9, 1.0]$ for AWGN, $[0.2, 0.4, 0.6]$ for fading.

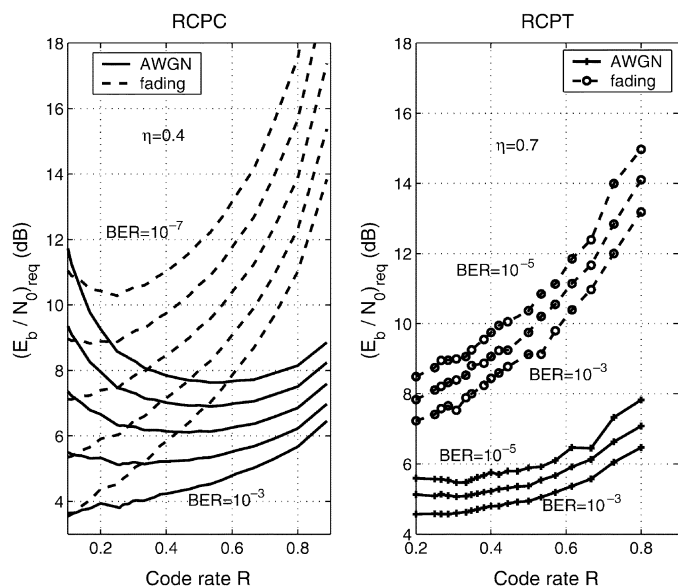


Fig. 6. Optimal signal-to-noise spectral density ratio, $(E_b/N_0)_{req}$, as a function of the target BER and code rate. RCPC: $\eta = 0.4$, $BER = [10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}]$. RCPT: $\eta = 0.7$, $BER = [10^{-3}, 10^{-4}, 10^{-5}]$.

V. CONCLUSIONS

The performance of RCPC/RCPT-coded DS-CDMA systems under a fixed-bandwidth expansion restriction is evaluated. Spectral efficiency and target optimal power are used as system performance measures for proper selection of the code rates. The performance of both MF and MMSE receivers over AWGN and Rayleigh fading channels are investigated.

It is shown that for the MF receiver, a lower code rate yields a better tradeoff. However, there is a limit, beyond which a further reduction in the code rate is no longer beneficial.

For the MMSE receiver under the same operating conditions, the best code rate for the RCPT codes is smaller than that for the RCPC codes. Also, the best code rate in a Rayleigh fading channel is smaller than that in an AWGN channel. These con-

clusions are inferred from inspection of the slope of the performance curves. It is also observed that with an increase in system load or E_b/N_0 , or with a more stringent target BER requirement, a higher rate code is recommended. In this case, bandwidth expansion favors spreading.

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