Signals and Systems I

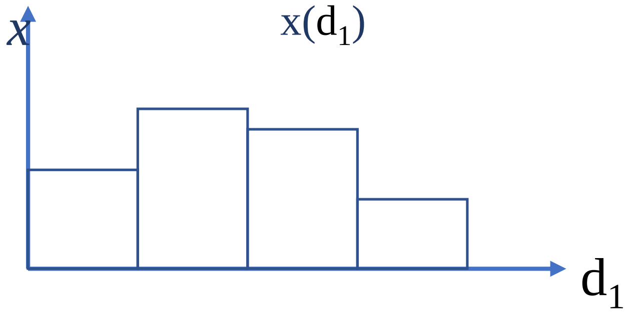
Topic 1

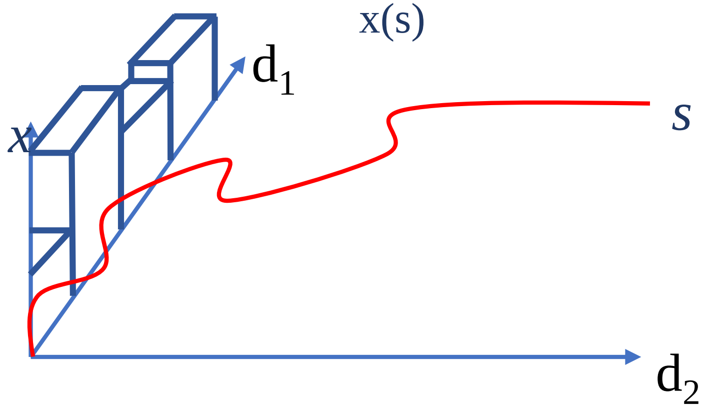
Today:

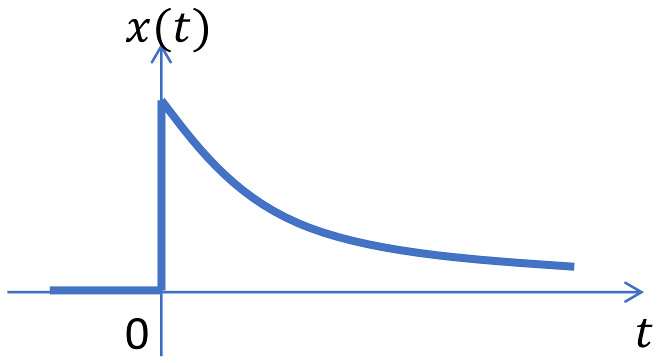
- Course Introduction and Outline
- What is Signal?
- What is System?
- Course Subjects
- Signal Classification
    1. Continuous Time (CT) Signals & Discrete Time (DT) Signals
    2. Analog Signals & Digital Signals
    3. Causal Signals
    4. Periodic Signals& Aperiodic Signals
    5. Energy Signals & Power Signals
- Important Signals

Signal : “ Function of independent variables that carry information”

Example: radio signals, electrical signals, biomedical signals (such as MRI)..







Independent Variables:

One dimensional Trajectories (d_1)

Two dimensional Trajectories (s) defined with (d_1, d_2)

One dimensional time (t)

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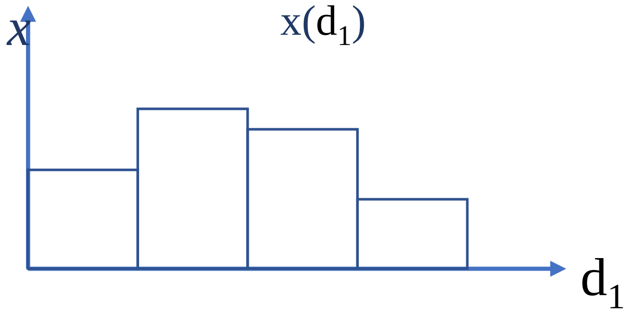

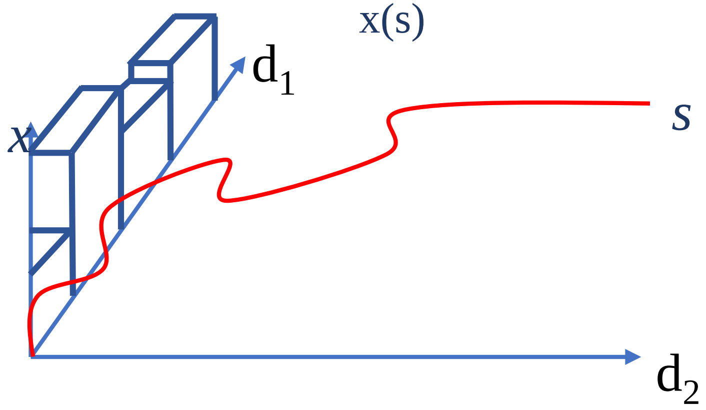

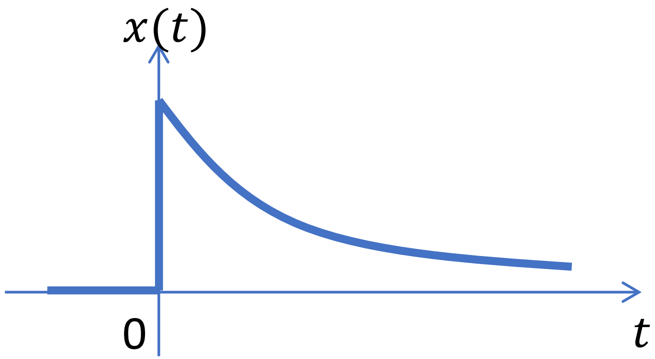

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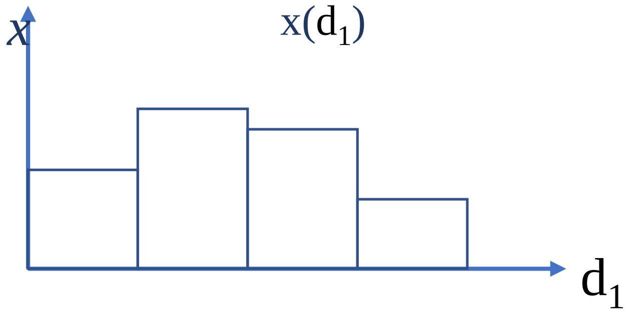

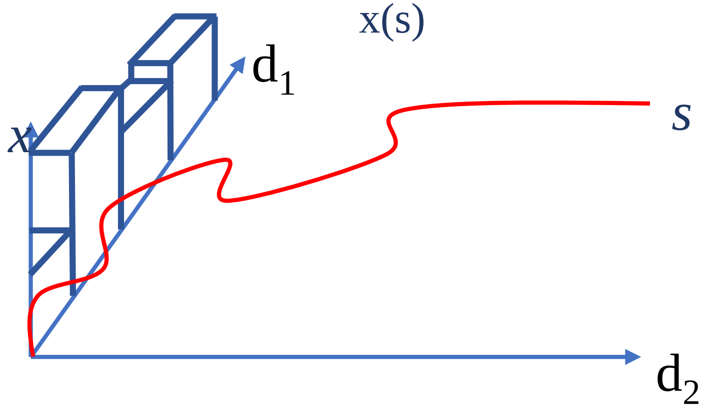

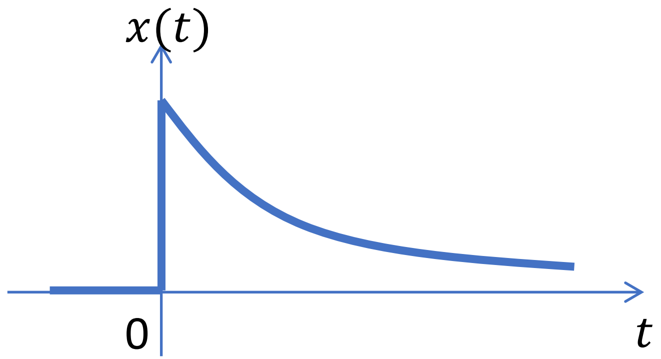

x(d1) = Height of buildings with respect to axis d1
x(s) = x(d1; d2) = Height of buildings on trajectory s.
x(t) as a function of time illustrating voltage or current

Signal : " Function of independent variables that carry information"

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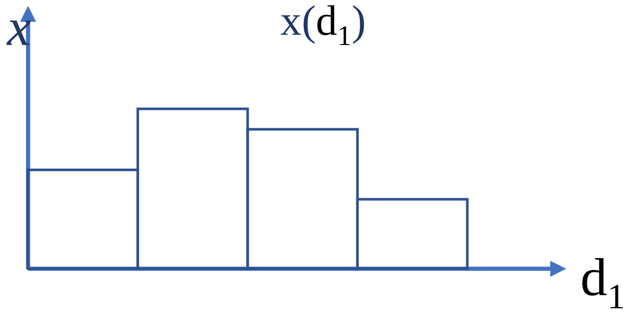
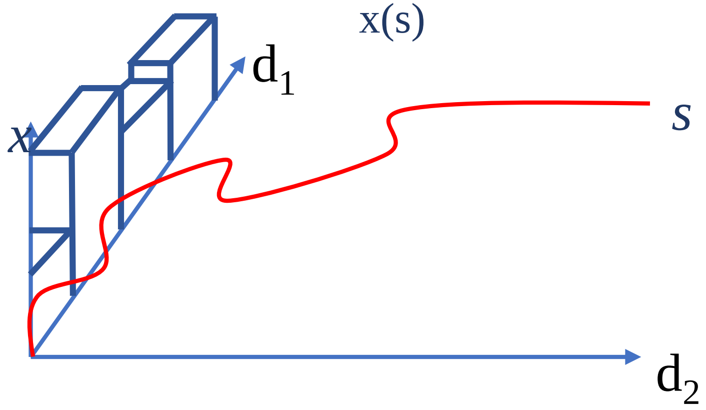
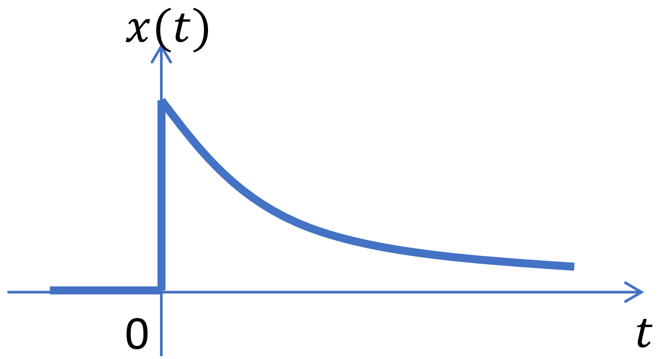






Information: Knowledge carried by the signal.

(defined by the signal observer)

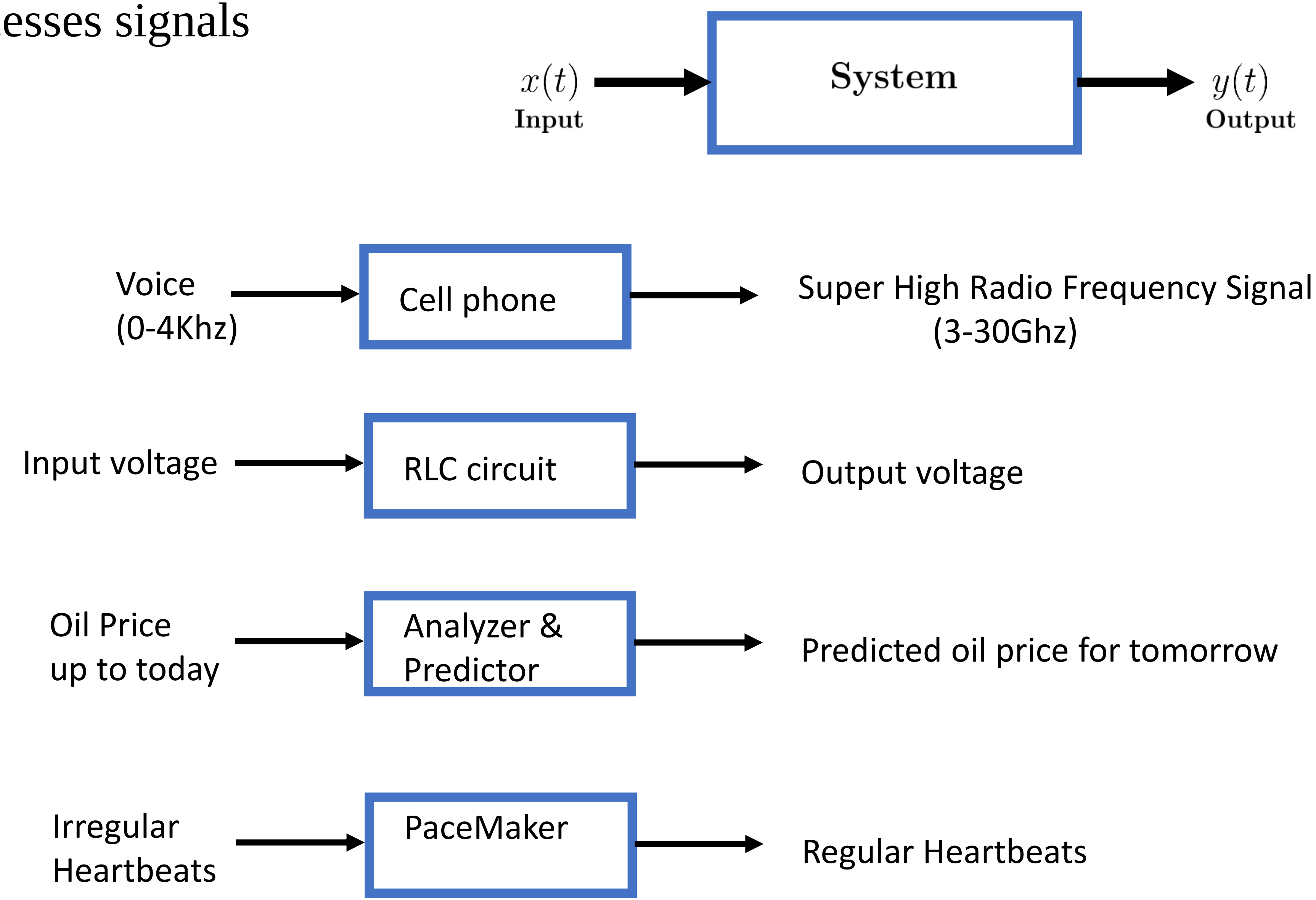
For example:

-height of the buildings for urban planning and Reconstructed Areas

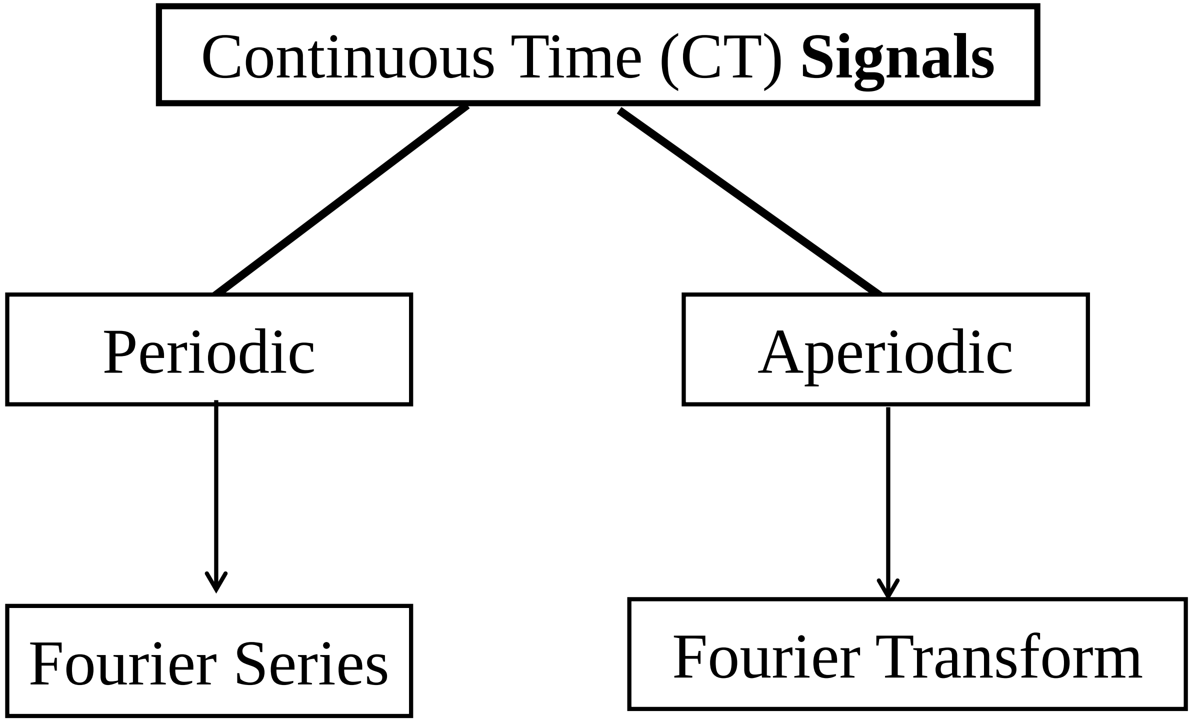
-heart beat in ECG for stroke prediction

- voltage, current or power for circuit analysis or design

Systems: Processes signals



Course Subjects:



<u>In Time Domain</u>

1. Signal Classification
2. Signal Operation
3. Important Signals

1

<u>In Frequency Domain</u>

1. Fourier Series
2. Fourier Transform

3

4

Course Subjects:

Continuous Time (CT) **Systems**

In Time Domain

1. Properties
2. Linear Time-Invariant (LTI) Systems
3. Impulse Response
4. Convolution

2

In Laplace and Frequency Domain

Laplace Transform & Frequency Analysis

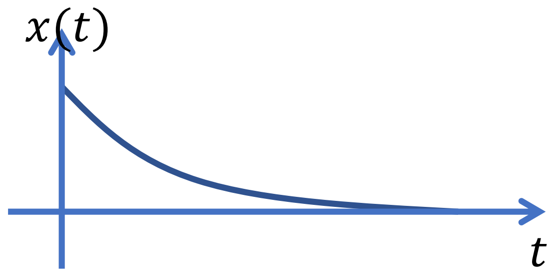
5

Signal Classification:

1-Continuous Time (CT) & Discrete Time (DT) Signals

- **CT:** Independent variable (t) is continuous
- **DT:** Independent variable (n) is discrete

(a)



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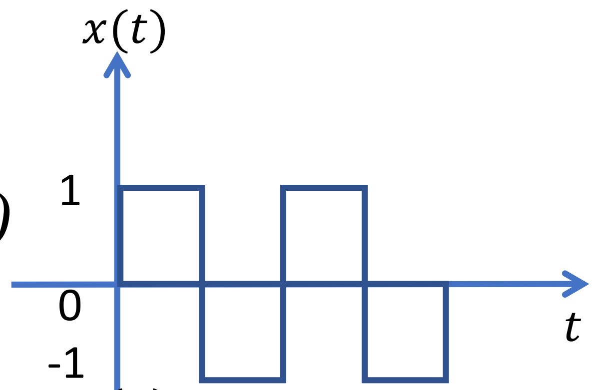
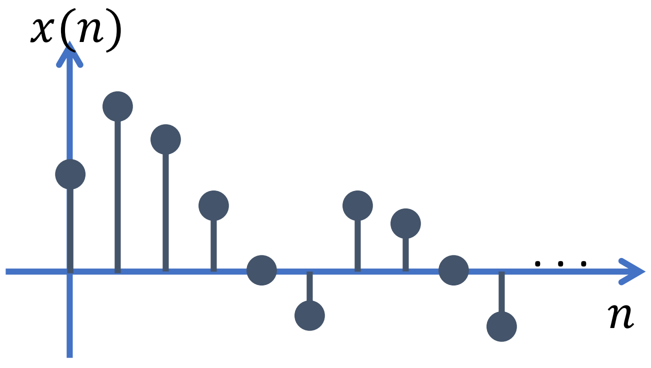


(b)



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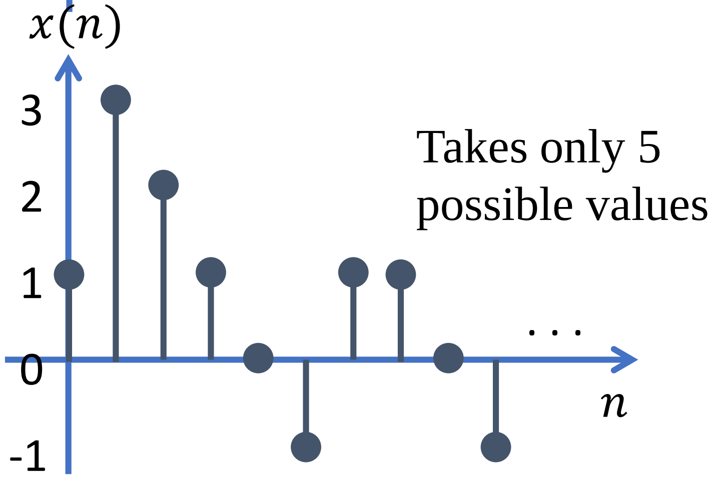

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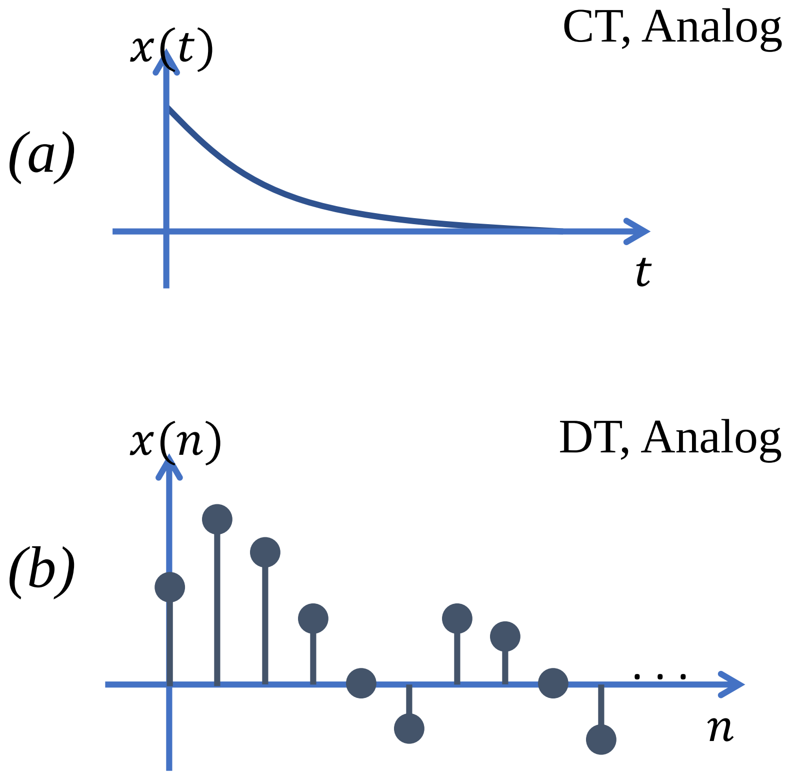
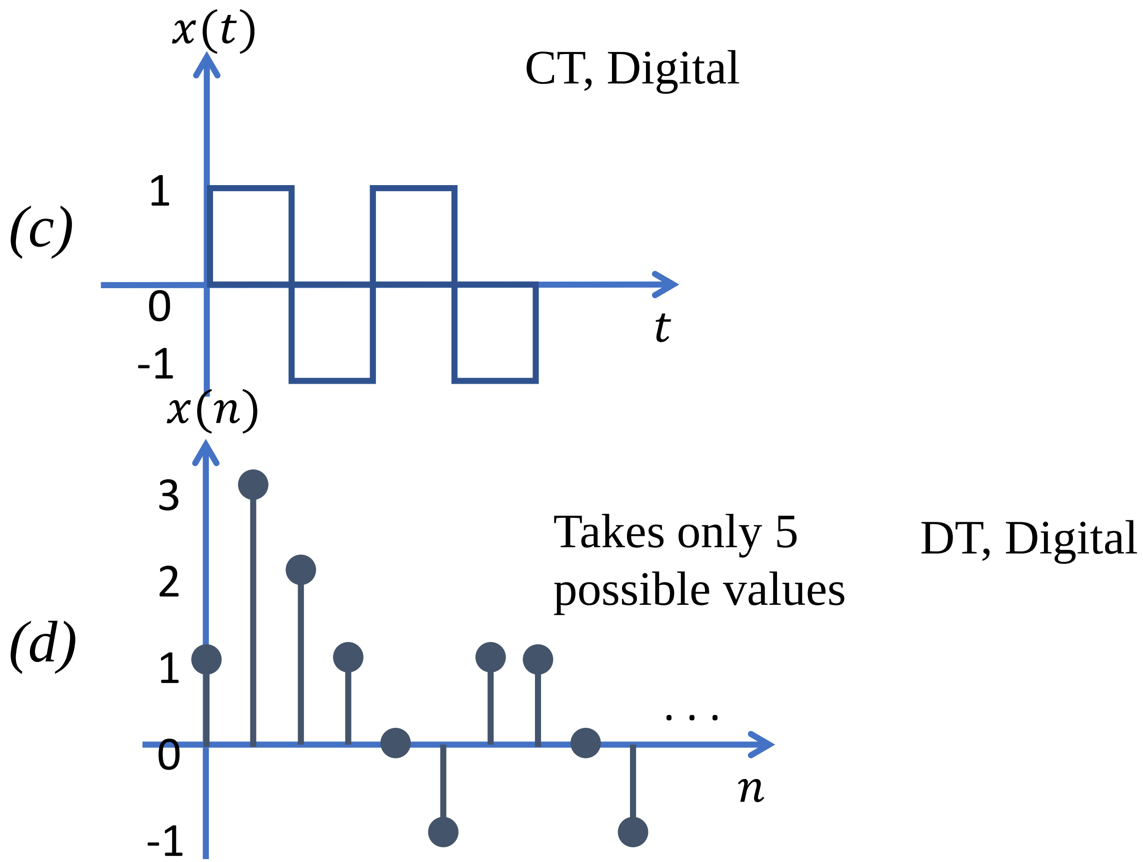




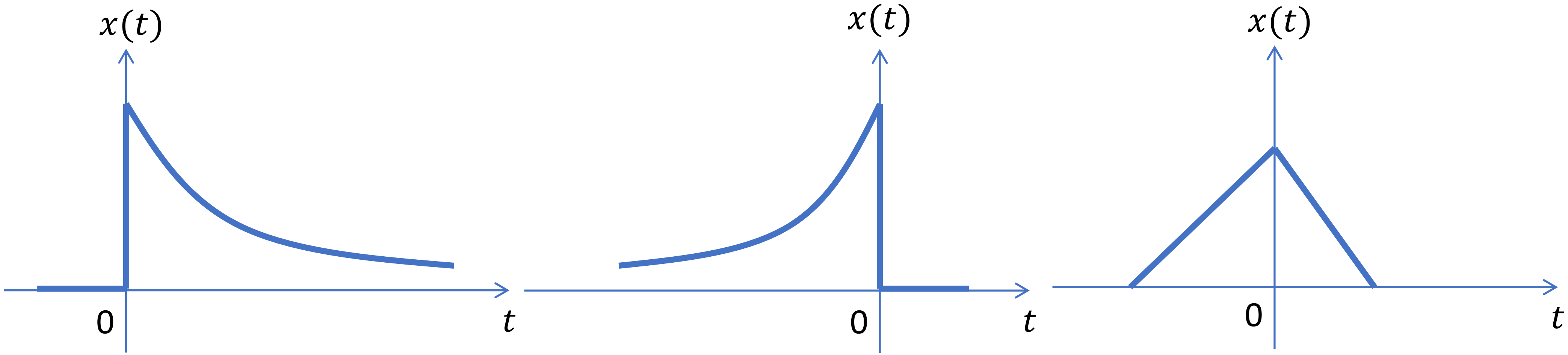
2-Analog and Digital Signals

- Analog Signal`s amplitude can take any value in a continuous range
- Digital Signal`s amplitude can take only a finite number of values

Signal Classification:

3-Causal Signals

- A signal that does not start before $t = 0$ is causal, $x(t) = 0, t < 0$
- A signal that starts before $t = 0$ is non-causal
- A signal that is zero for all $t \geq 0$ is anti-causal $x(t) = 0, t > 0$



Signal Classification:

3-Causal Signals

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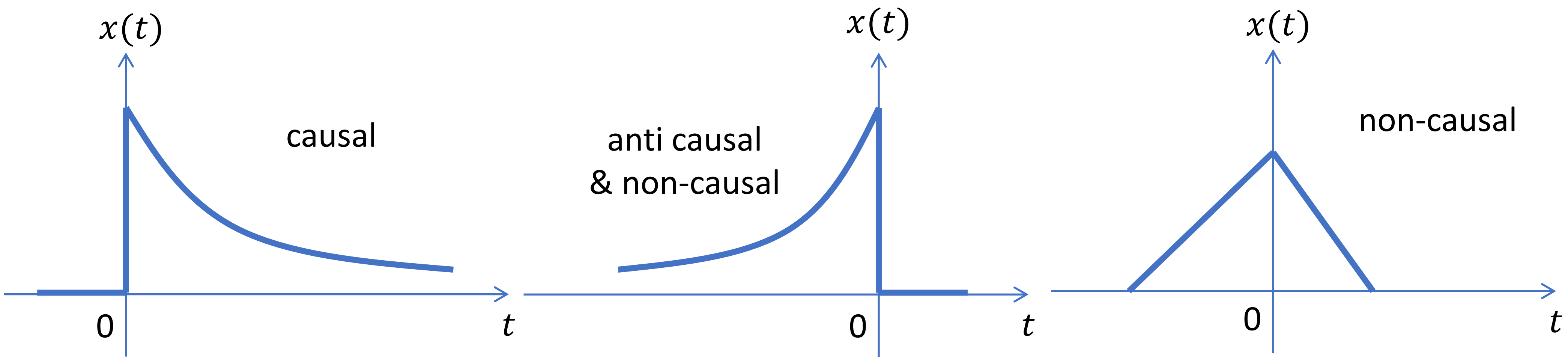


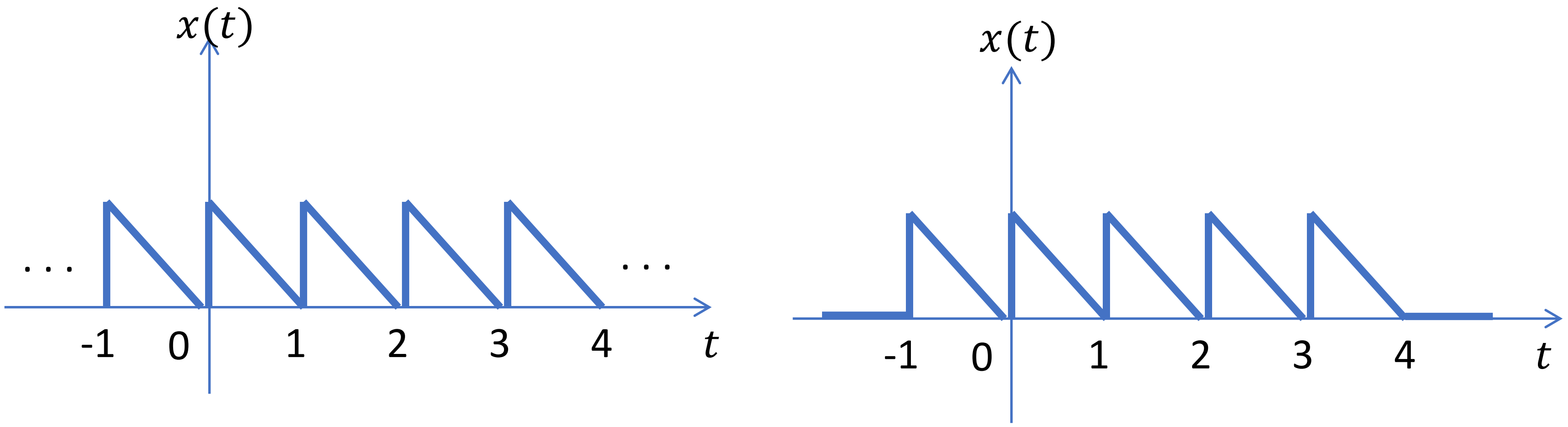
Signal Classification:

4-Periodic and Aperiodic Signals

$x(t)$ is periodic if for some positive constant T_0 we have the following relation:

$$x(t) = x(t + T_0) \text{ for all } t.$$

The smallest value of T_0 is the fundamental period.



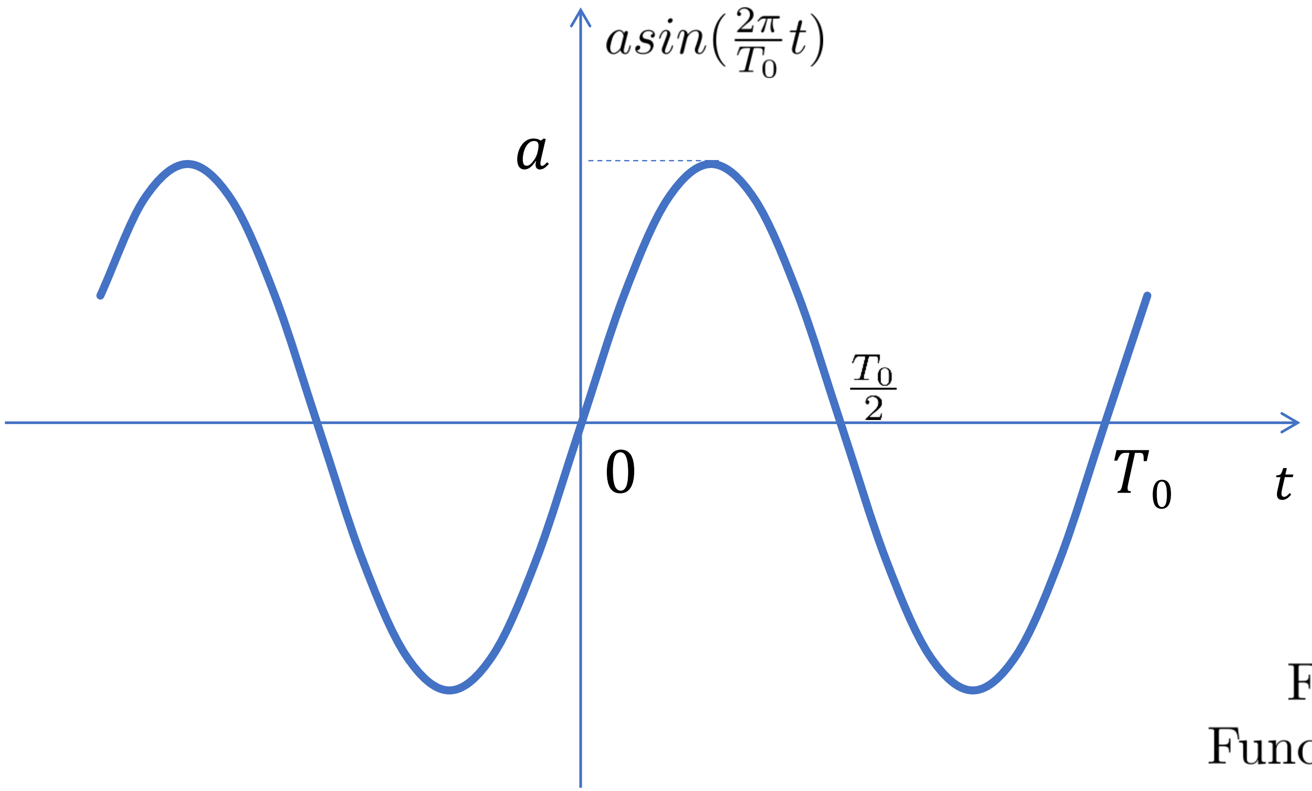
Note: Periodic signals are Non-Causal.

Any signal that is not periodic is aperiodic

Signal Classification:

4-Periodic and Aperiodic Signals

Reminder of fundamental period of sine waves:

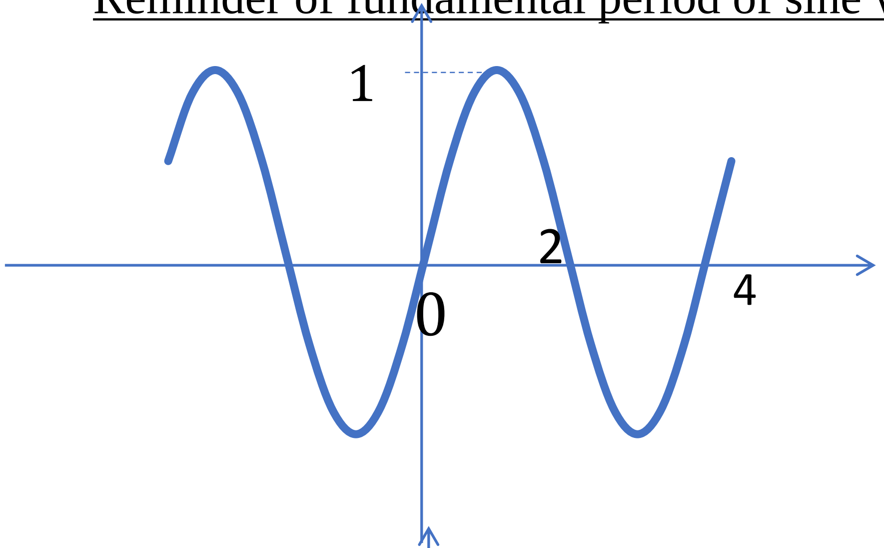


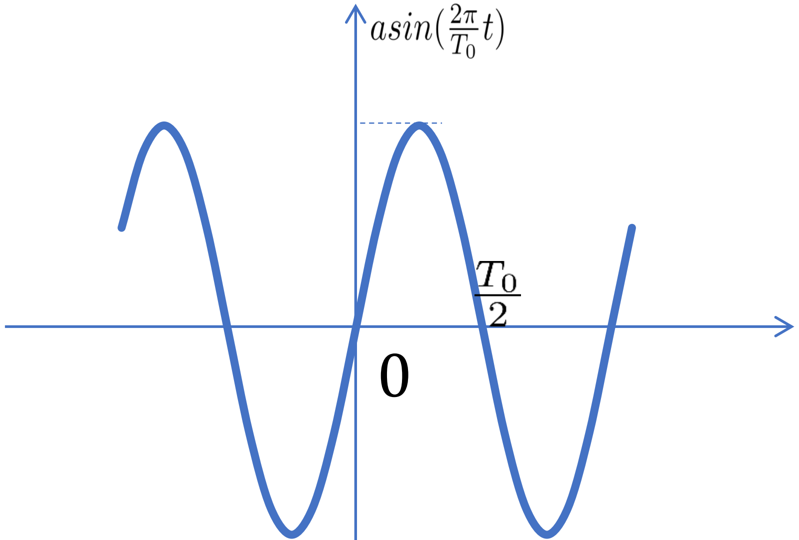
$$\text{Fundamental Period: } T_0$$
$$\text{Fundamental Freq: } w = \frac{2\pi}{T_0}$$

Signal Classification:

4-Periodic and Aperiodic Signals

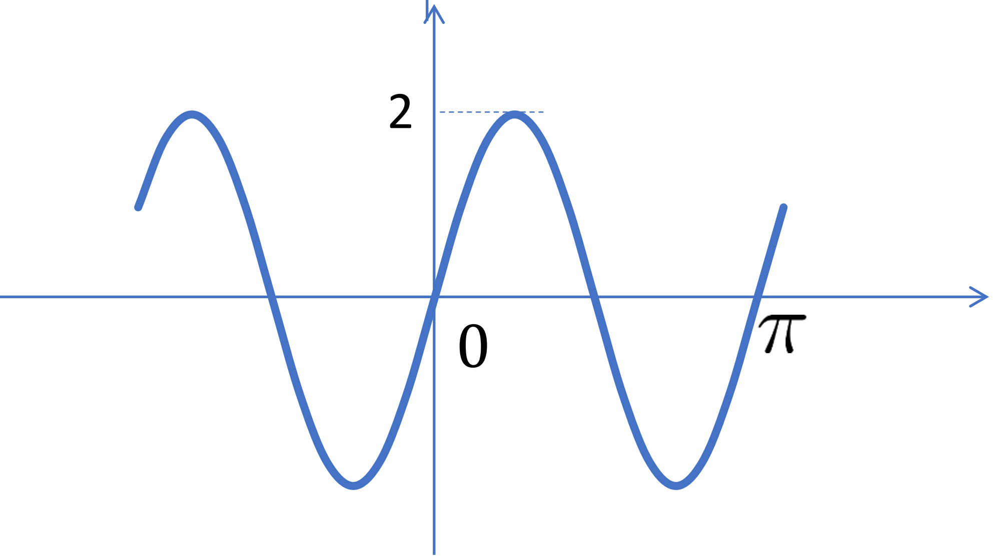
Reminder of fundamental period of sine waves:

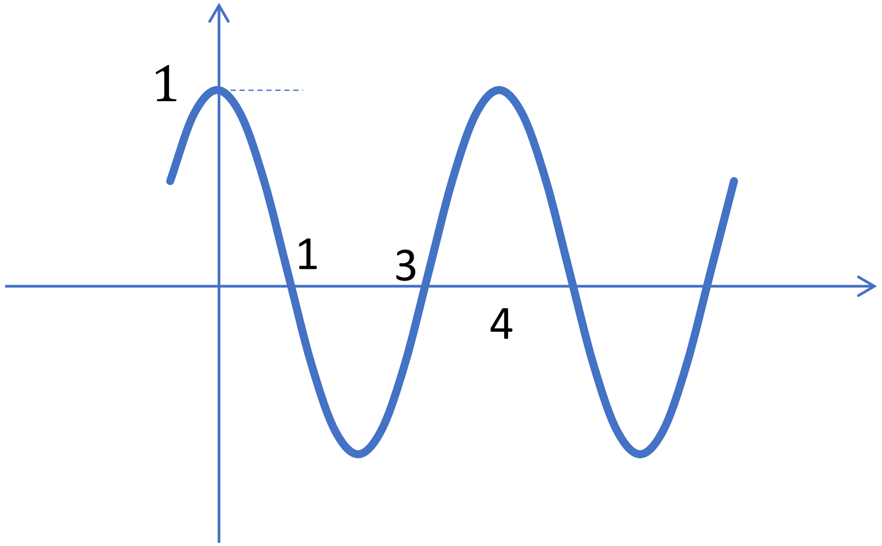




Fundamental Period: T_0

Fundamental Freq: $w = \frac{2\pi}{T_0}$





Signal Classification:

5-Energy Signal and Power Signal

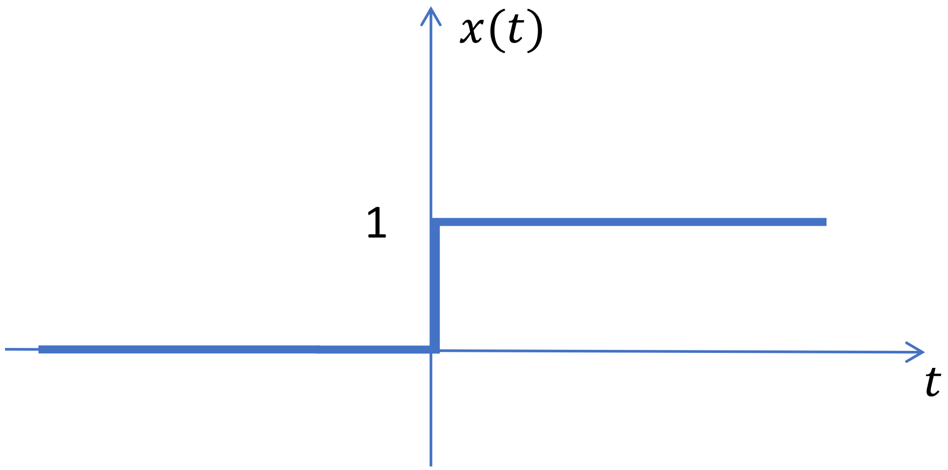
- Signal's Energy: $E_x = \int_{-\infty}^{\infty} \mid x(t) \mid^2 \, dt$ (force energy unit is Joule (J))
- Signal's Power: $P_x = \lim_{T \to \infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 \, dt$ (power unit: Watt=Joule per second (W=J/s)
- r.m.s (root mean square) = $\sqrt{P_x}$
- A signal with finite energy is an <u>energy signal</u>
- A signal with finite and non-zero power is a <u>power signal</u>

Signal Classification:

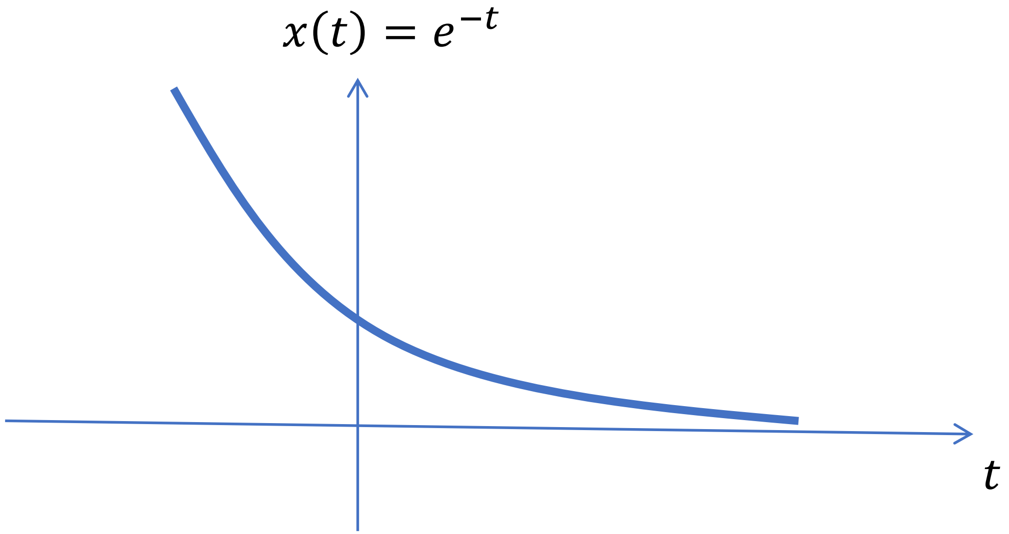
5-Energy Signal and Power Signal

Which of the following signals are energy or power signal?

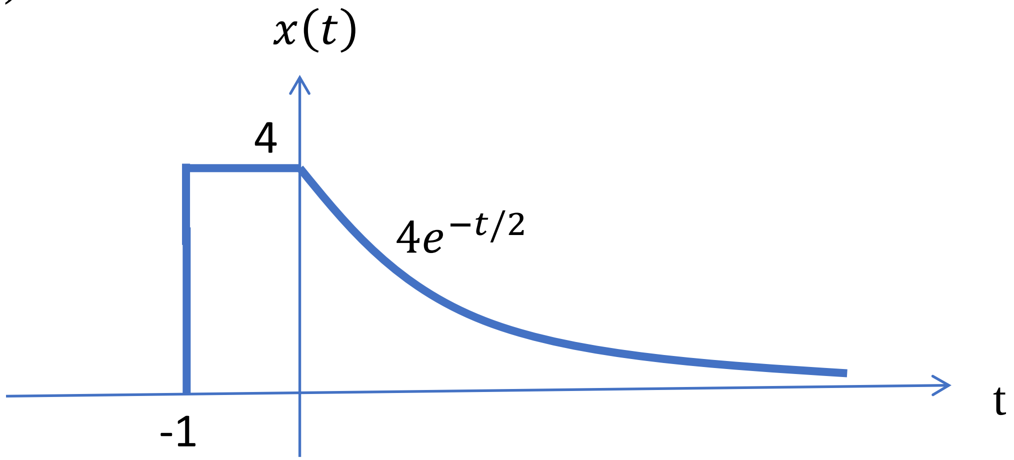
a)



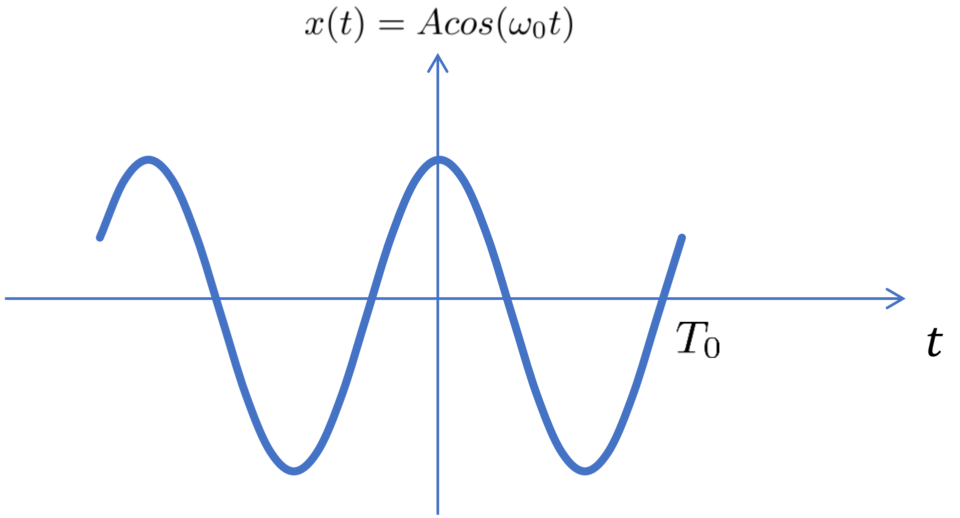
b)



c)

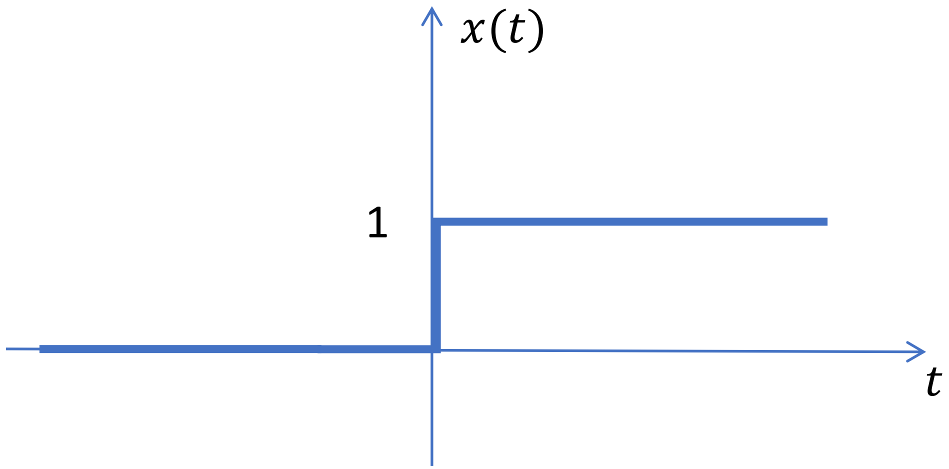


d)



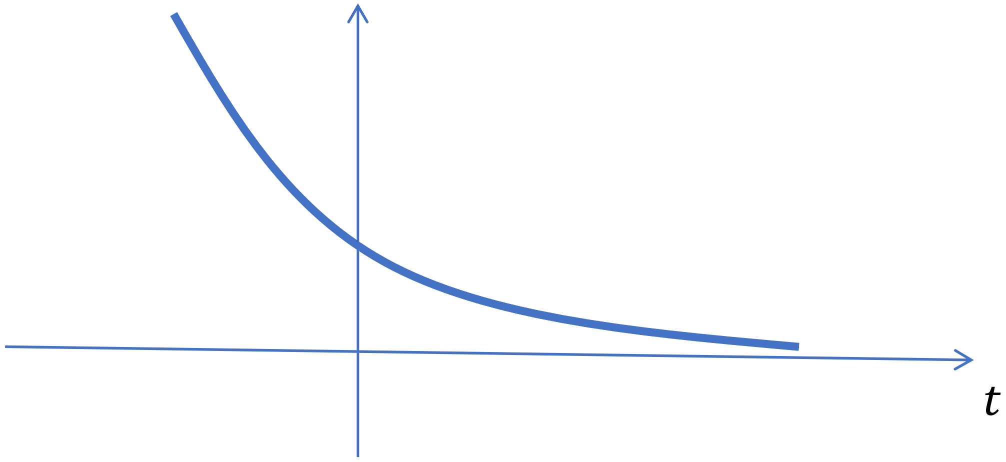
Calculating the Energy: $E_x = \int_{-\infty}^{\infty} \mid x(t) \mid^2 dt$

a)



b)

$x(t) = e^{-t}$



(a) $\int_{-\infty}^{\infty} \mid x(t) \mid^2 dt = \int_0^{\infty} 1dt = \infty$! not energy signal

(b) $\int_{-\infty}^{\infty} \mid x(t) \mid^2 dt = \int_{-\infty}^{\infty}(e^{-t})^2 dt = \int_{-\infty}^{\infty}(e^{-2t})dt = -\frac{1}{2}e^{-2t}\Big|_{-\infty}^{\infty} =$

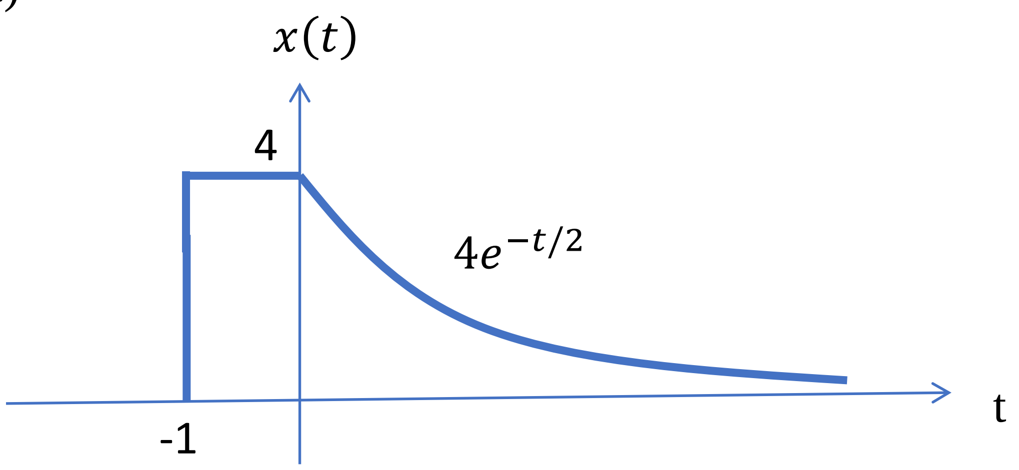
$0 - (-\infty) = \infty$! Not energy signal

Reminder: $\int e^{\alpha t} = \frac{1}{\alpha}e^{\alpha t}$

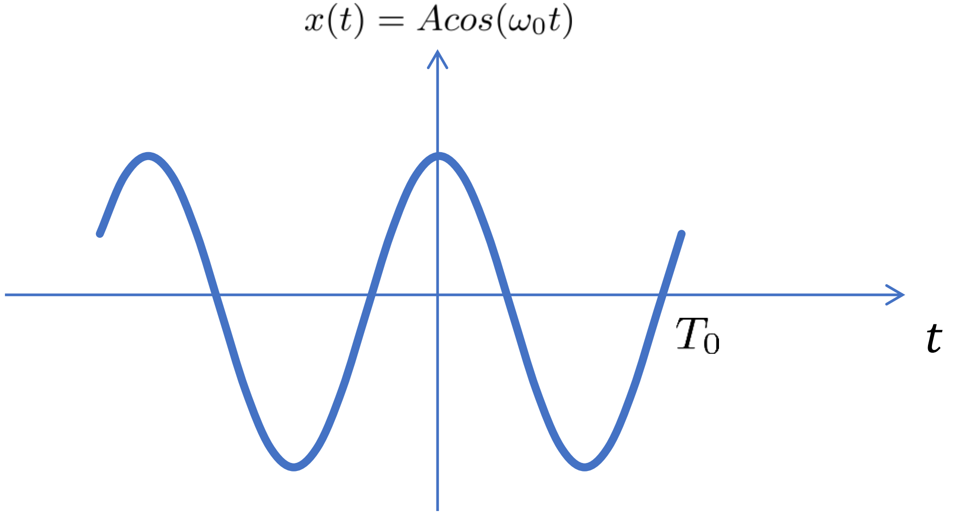
Calculating the Energy:

$$E_x = \int_{-\infty}^{\infty} \mid x(t) \mid^2 dt$$

c)



d)

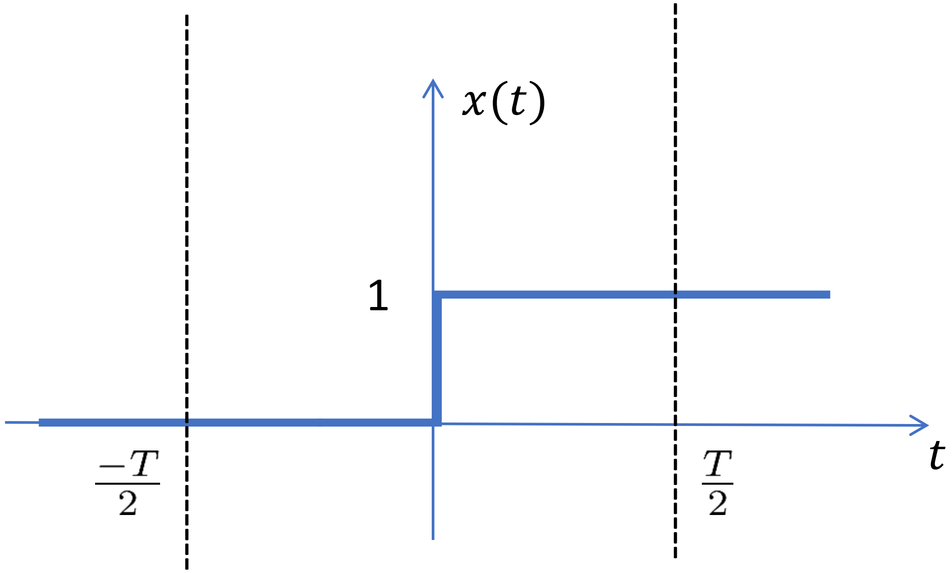


(c) $\int_{-\infty}^{\infty} \mid x(t) \mid^2 dt = \int_{-1}^{0} (4)^2 dt + \int_{0}^{\infty} 16e^{-t} dt = 16(0-(-1)) + 16 \frac{e^{-t}}{-1}\Big|_0^{\infty} =$
$16 + 16 = 32$ Energy Signal

(d) $\int_{-\infty}^{\infty} \mid x(t) \mid^2 dt = \infty$ Always for periodic signals (why?)

Calculating the Power:

a)



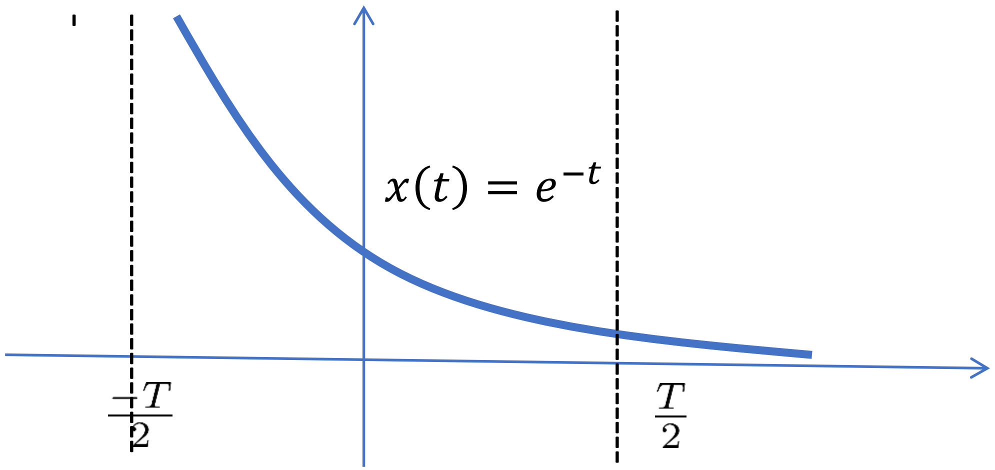
$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt$$

$$\text{(a)} \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt = \lim_{T\to\infty} \left(\frac{1}{T} \int_{-\frac{T}{2}}^{0} \mid x(t) \mid^2 dt + \frac{1}{T} \int_{0}^{\frac{T}{2}} \mid x(t) \mid^2 dt \right)$$

$$= \lim_{T\to\infty} \frac{1}{T}(0 + \frac{T}{2}) = \frac{1}{2}. \quad \text{This is a power signal.}$$

Calculating the Power:

b)



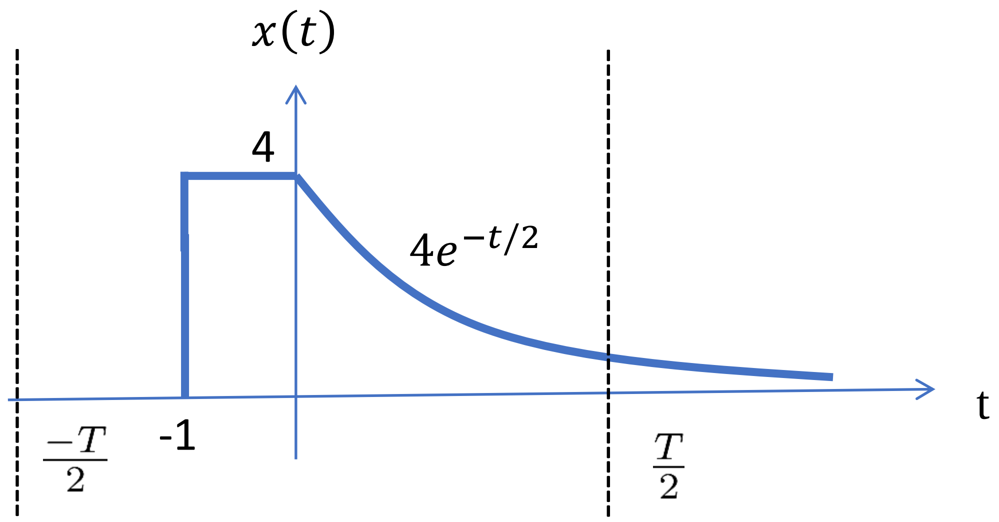
$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt$$

(b) $\lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} e^{-2t} dt = \lim_{T\to\infty} \frac{1}{T} \left(\frac{e^{-2t}}{-2} \right) \Big|_{\frac{-T}{2}}^{\frac{T}{2}} = \lim_{T\to\infty} \frac{-1}{2T} \left(e^{-T} - e^{T} \right) = \lim_{T\to\infty} \left(\frac{e^{T} - e^{-T}}{2T} \right) =$

(From L' Hopital rule)$= \lim_{T\to\infty} \left(\frac{e^{T} + e^{-T}}{2} \right) = \infty$ This is not a power signal.

Calculating the Power:

c)



$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt$$

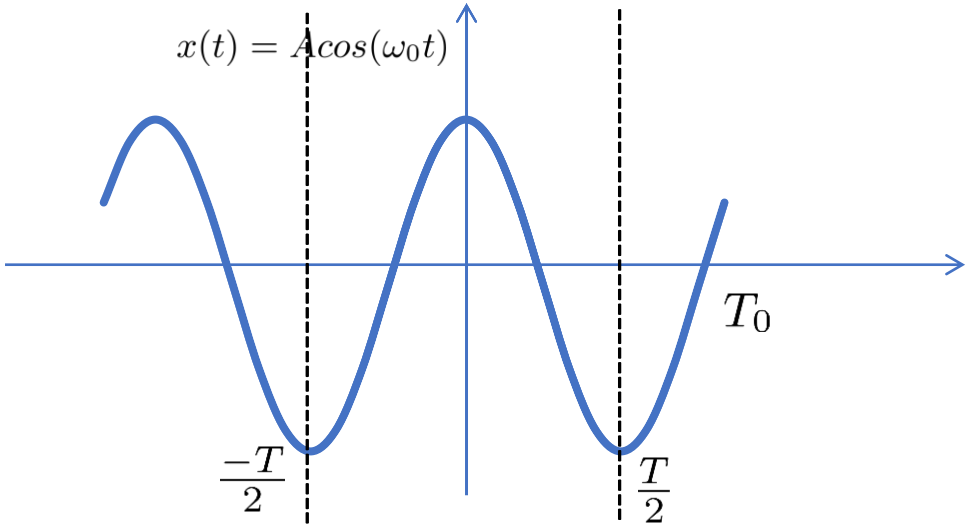
$$\text{(c)}\ \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt = \frac{1}{T} \int_{\frac{-T}{2}}^{-1} |x(t)|^2 dt + \frac{1}{T} \int_{-1}^{0} \mid x(t) \mid^2 dt + \frac{1}{T} \int_{0}^{\frac{T}{2}} \mid x(t) \mid^2$$

$$dt = \frac{1}{T} \left(0 + 16 + 16 \left(\frac{e^{-t}}{-1} \right) \Bigg|_0^{\frac{T}{2}} \right) = \frac{1}{T} \left(16 - 16e^{-\frac{T}{2}} + 16 \right) = \frac{32-16e^{-\frac{T}{2}}}{T}$$

$$P_x = \lim_{T\to\infty} \frac{32-16e^{-\frac{T}{2}}}{T} = \lim \frac{32}{\infty} = 0. \quad \text{Not a power signal}$$

Calculating the Power:

d)



$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt$$

(d) Power of a periodic signal$=\frac{1}{T_0}$(energy of one period)

$$x(t) = Acos(\omega_0 t)$$

$$|x(t)|^2 = A^2 cos^2(\omega_0 t) = A^2 \left(\frac{1+cos(2\omega_0 t)}{2}\right)$$

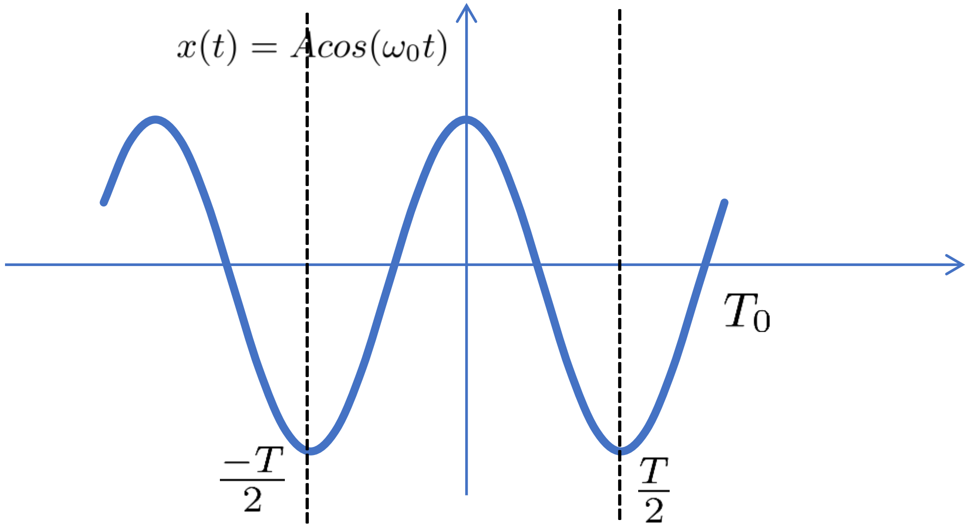
$$P_x = \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2 dt = \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} A^2 \left(\frac{1+cos(2\omega_0 t)}{2}\right) dt = \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} \frac{A^2}{2} dt + \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} \frac{cos(2\omega_0 t)}{2} dt$$

$$= \frac{1}{T_0} \frac{A^2}{2} T_0 + 0 = \frac{A^2}{2} \text{ Power Signal}$$

Calculating the Power:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} \mid x(t) \mid^2 dt$$

d)



For periodic signals with fundamental period T_0 we have

$$P_x = \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} \mid x(t) \mid^2 dt$$

Note that the integral can be taken over any interval of length T_0.

(d) Power of a periodic signal$=\frac{1}{T_0}$(energy of one period)

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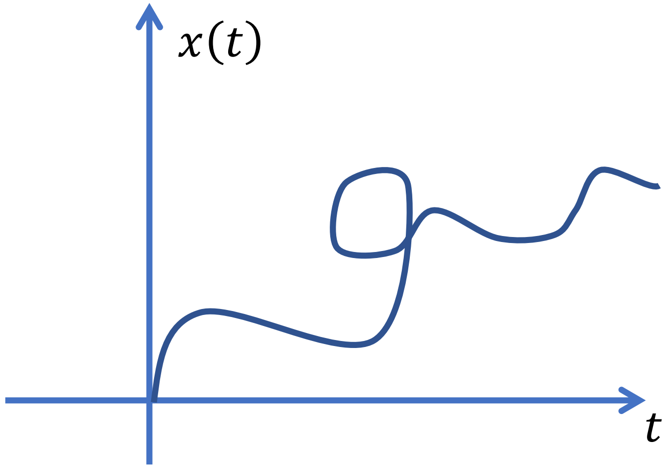
$$= \frac{1}{T_0} \frac{A^2}{2} T_0 + 0 = \frac{A^2}{2} \text{ Power Signal}$$

Question: Is the following graph a signal?



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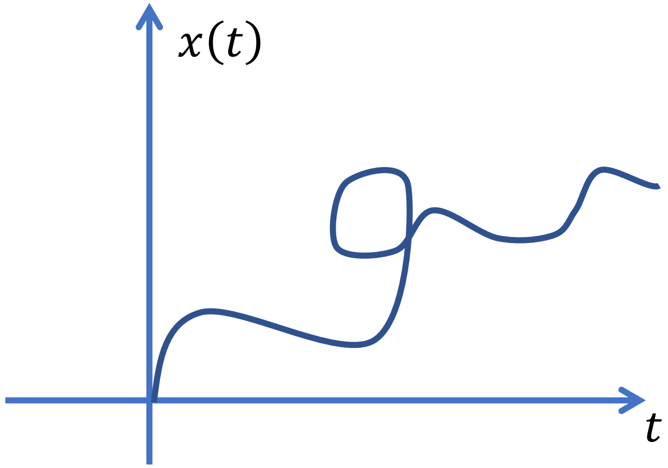


Answer: The above graph is NOT a signal. For the function $x(t)$ to be a signal, there must be no more than one value of $x(t)$ at any given time t.

Some Important Signals:

$u(t)$: unit step function

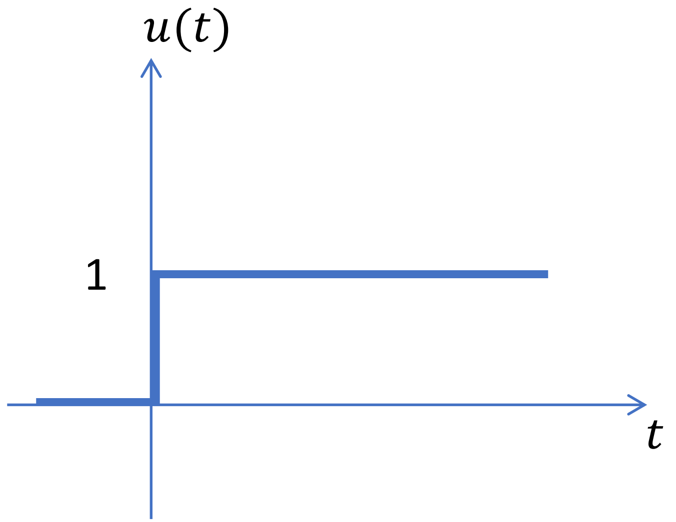
$\delta(t)$: unit delta function

Sinc function

e^{st}: exponential function

Some Important Signals:

$u(t)$: unit step function

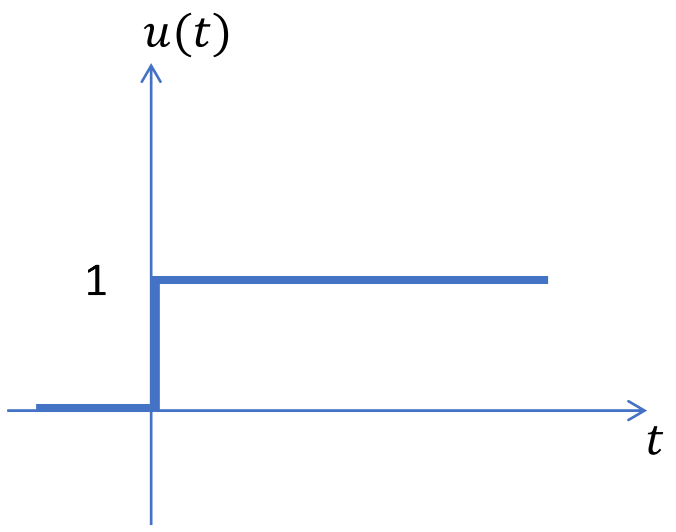


$$u(t) = \begin{cases} 1, & \text{if} \quad t \geq 0 \\ 0, & \text{if} \quad t < 0 \end{cases}$$

Question: Why is step function important?

Some Important Signals:

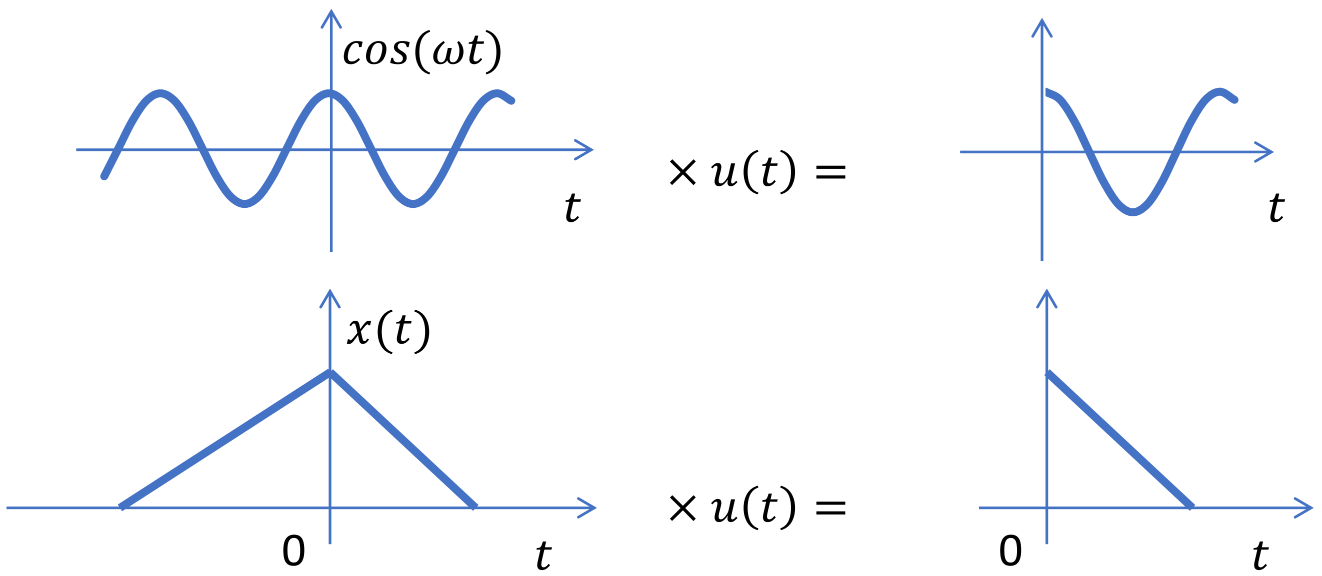
$u(t)$: unit step function



$$u(t) = \begin{cases} 1, & \text{if} \quad t \geq 0 \\ 0, & \text{if} \quad t < 0 \end{cases}$$

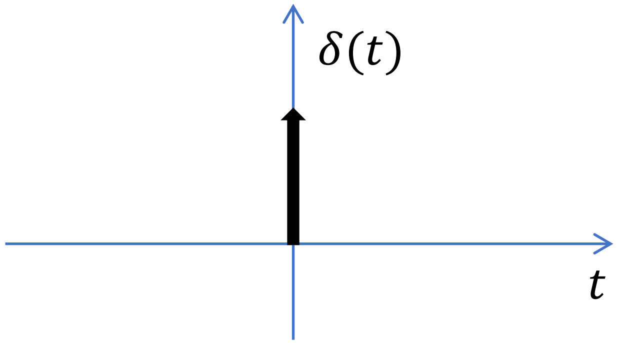
Question: Why is step function important?

- To define a sudden jump
- To define the causal part of a signal
- To define segment & more (later)



Some Important Signals:

$$\delta(t)\text{: unit impulse function, Dirac's Delta function}$$



$$\delta(t) = \begin{cases} 0, & \text{if} \quad t > 0 \\ 0, & \text{if} \quad t < 0 \\ \infty, & \text{if} \quad t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

By Paul Dirac founder of Quantum Physics

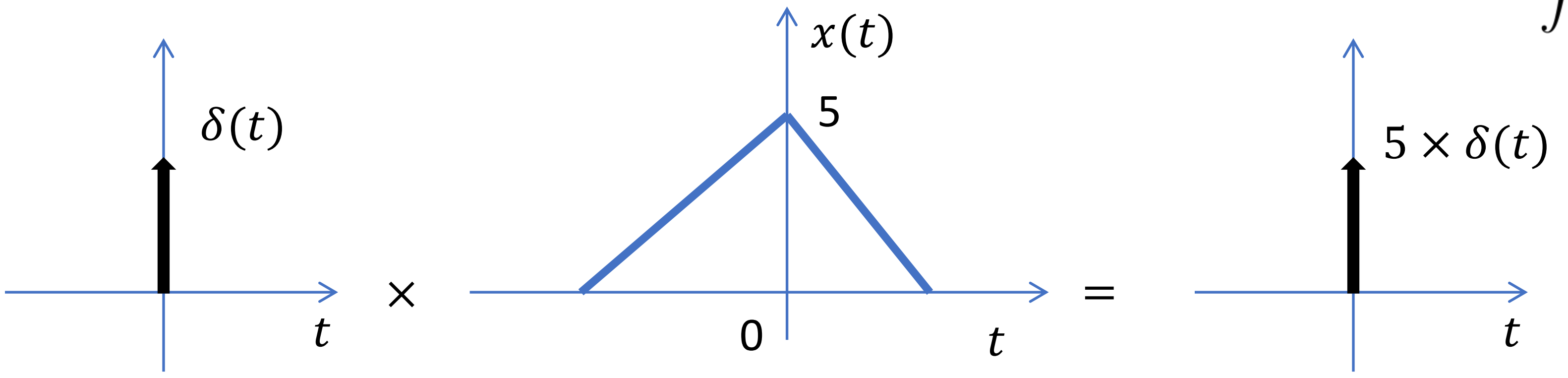
History: Paul Dirac was looking for a function to represent and model the density of an idealize point mass or point charge. Having point mass or point charge of zero everywhere except at zero should have been the property of this function. Since a function with a property as such was not defined yet, he introduced Dirac delta function.

Question: Why is $\delta(t)$ important?

$$\delta(t) = \begin{cases} 0, & \text{if} \quad t > 0 \\ 0, & \text{if} \quad t < 0 \\ \infty, & \text{if} \quad t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- $\delta(t)$ is derivative of $u(t)$: $\int_{-\infty}^{t} \delta(t)dt = u(t)$
- $\delta(t)x(t) = \delta(t)x(0)$



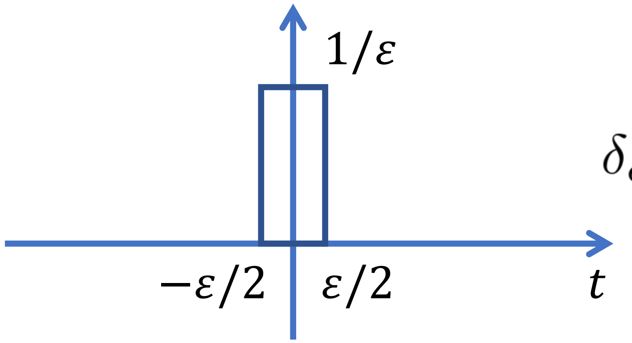
- $\int_{-\infty}^{\infty} \delta(t)x(t)dt = \int_{-\infty}^{\infty} \delta(t)x(0)dt = x(0)\int_{-\infty}^{\infty} \delta(t)dt = x(0)$
- In general for $(a > 0)$

$$\int_{-a}^{0^-} \delta(t)f(t)dt = 0, \quad \int_{-a}^{a} \delta(t)f(t)dt = f(0),$$

$$\int_{0^+}^{a} \delta(t)f(t)dt = 0, \quad \int_{0^-}^{0^+} \delta(t)f(t)dt = f(0)$$

Visualization of $\delta(t)$

- Very large at zero & zero at all non-zero points!
- Also $\int_{-\infty}^{\infty} \delta(t)dt = 1$
- We can consider the following box function:



$$\delta_\epsilon(t) = \begin{cases} 0, & \text{if} \quad t < \frac{-\epsilon}{2} \\ \frac{1}{\epsilon}, & \text{if} \quad \frac{-\epsilon}{2} < t < \frac{\epsilon}{2} \\ 0, & \text{if} \quad t > \frac{\epsilon}{2} \end{cases}$$

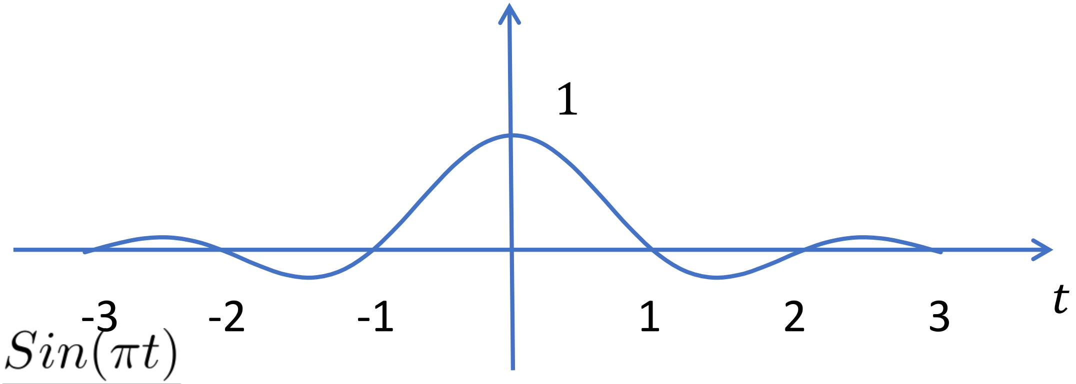
$$\int_{-\infty}^{\infty} \delta_\epsilon(t)dt = 1$$

$$limit_{\epsilon \to 0} \delta_\epsilon(t) = \delta(t)$$

- note that the following information and visualization of the delta function is only an abstract. There is no true visualization or illustration for $\delta(t)$, for the same reason that the value of *0* or *infinity* are abstracts and not truly definable.

Some Important Signals:

Sinc Function



>> Sinc

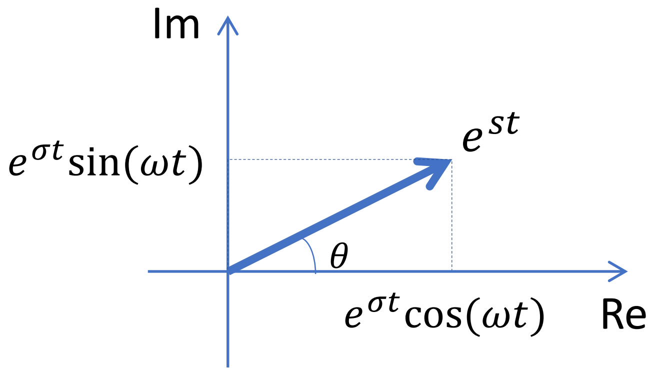
- Definition in Matlab: $Sinc(t) = \frac{Sin(\pi t)}{\pi t}$
- $Sinc(0) = \lim_{t \to 0} \frac{Sin(\pi t)}{\pi t} = \frac{\pi cos(\pi t)}{\pi} = 1$
- $\int_{-\infty}^{\infty} Sinc(t)dt = 1$

Definition in some books: $Sinc(t) = \frac{Sin(t)}{t}$

One Very Important Signals:

e^{st}: exponential function

$$x(t) = e^{st} \begin{cases} e = 2.71828... & \frac{de^x}{dx} = e^x \\ s = \sigma + j\omega & \sigma = \text{real part of } s \quad \omega = \text{imaginary part of } s. \end{cases}$$

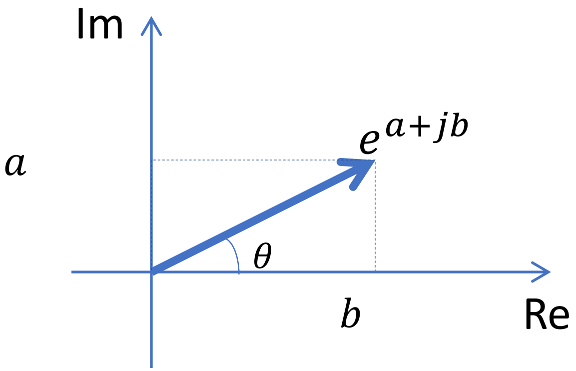


$$s = \sigma + j\omega$$

$$\begin{aligned} e^{st} &= e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} \\ &= e^{\sigma t}\,(cos\omega t + jsin\omega t) \\ &= \underbrace{e^{\sigma t}\cos(\omega t)}_{Real\ part} + j\ \underbrace{e^{\sigma t}\sin(\omega t)}_{Imaginary\ part} \end{aligned}$$

$$|a + jb| = \sqrt{a^2 + b^2}$$

Euler identity formula: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$



$$|e^{st}|^2 = |e^{\sigma t}e^{j\omega t}|^2 = |e^{\sigma t}|\underbrace{|e^{j\omega t}|^2}_{1} = e^{2\sigma t}$$

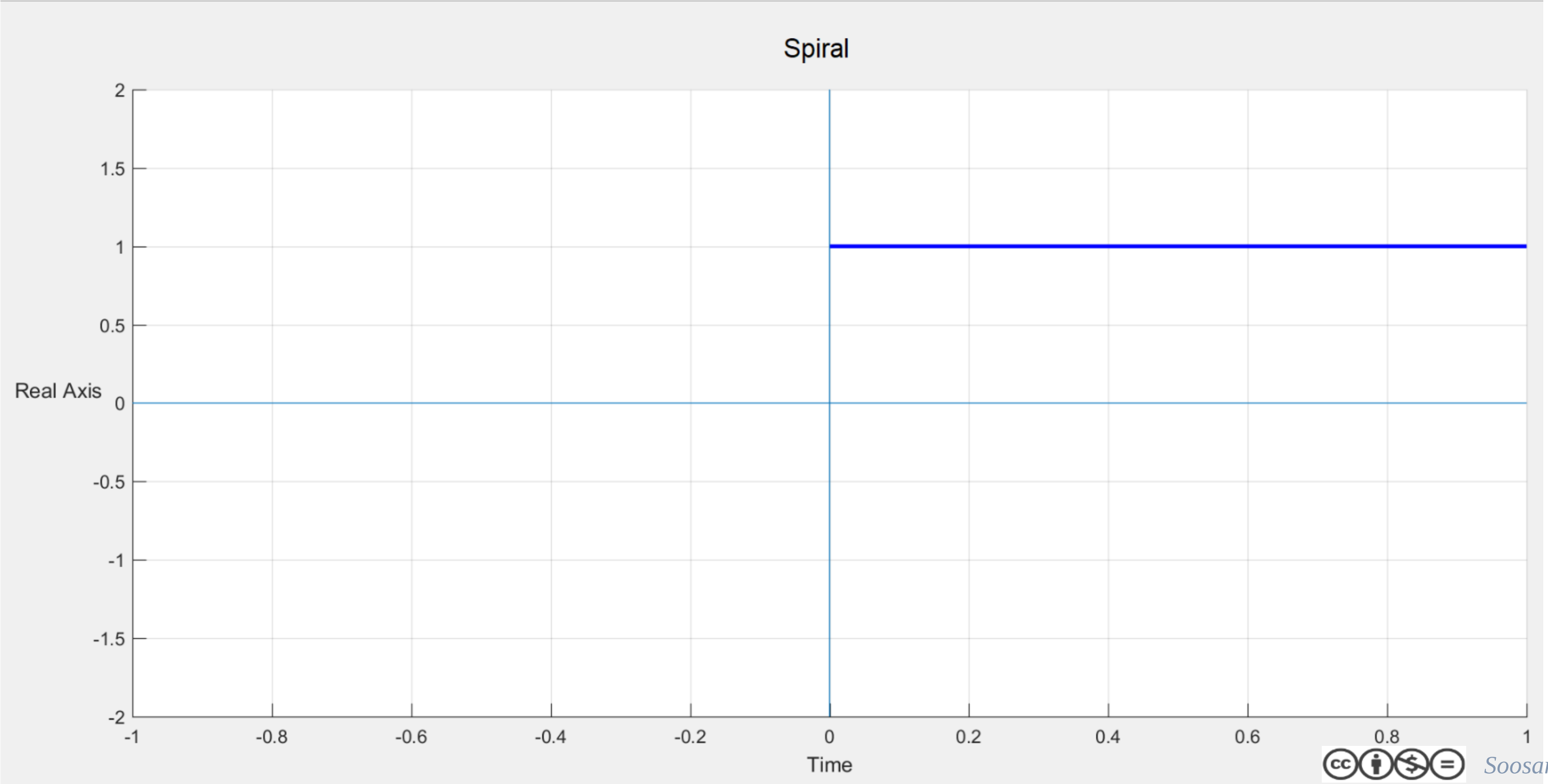
$$|e^{st}|^2 = |\underbrace{e^{\sigma t}cos(\omega t)}_{a} + j\,\underbrace{e^{\sigma t}sin(\omega t)}_{b}|^2 = e^{2\sigma t}\underbrace{\left(cos^2(\omega t) + sin^2(\omega t)\right)}_{1} = e^{2\sigma t}$$

Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

$1.\ \ s = 0,\ \ e^{0t} = 1$

Note that graphs in this page and the following pages show the functions for positive time only. The signals have values for negative time as well.



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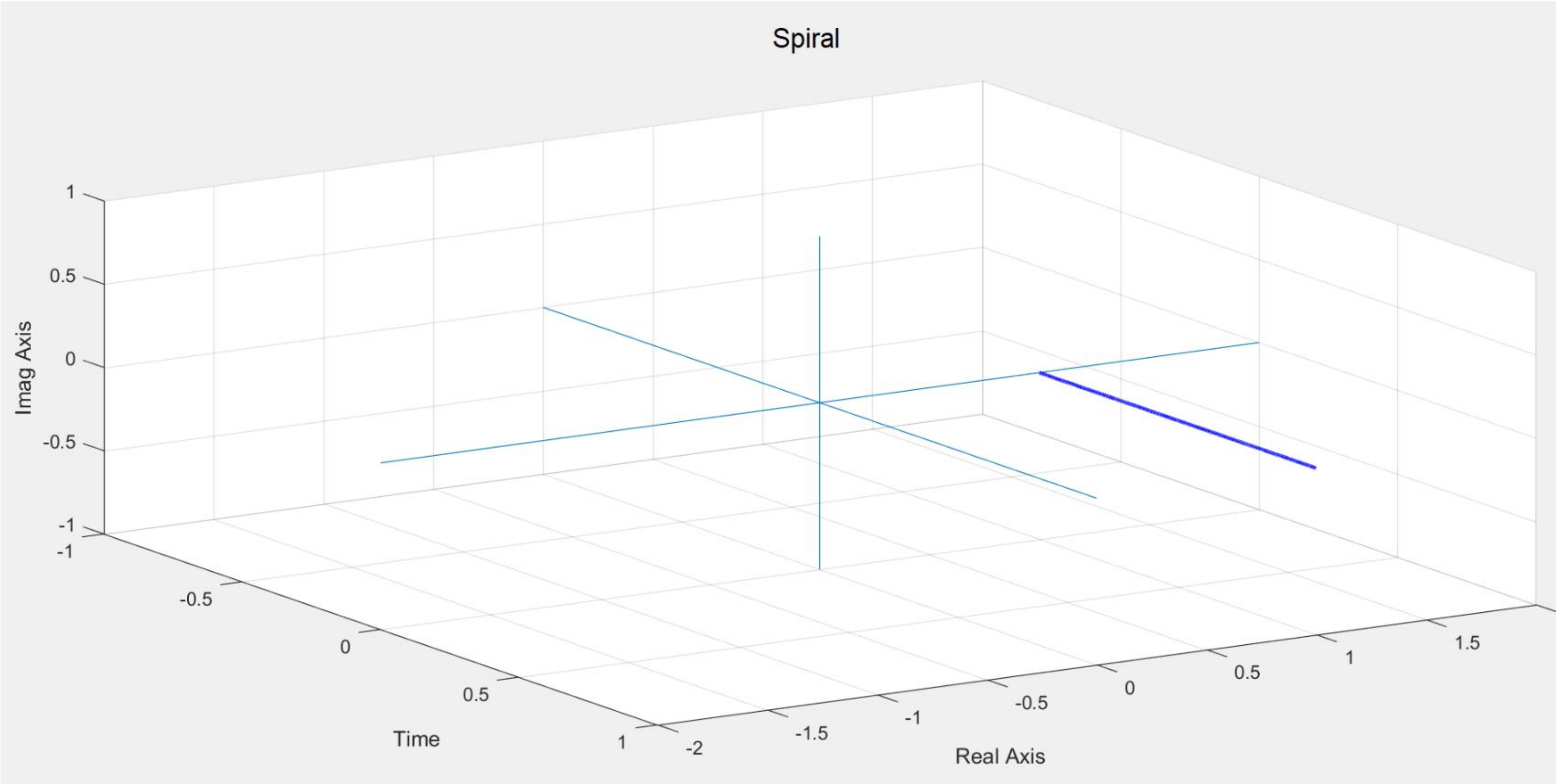
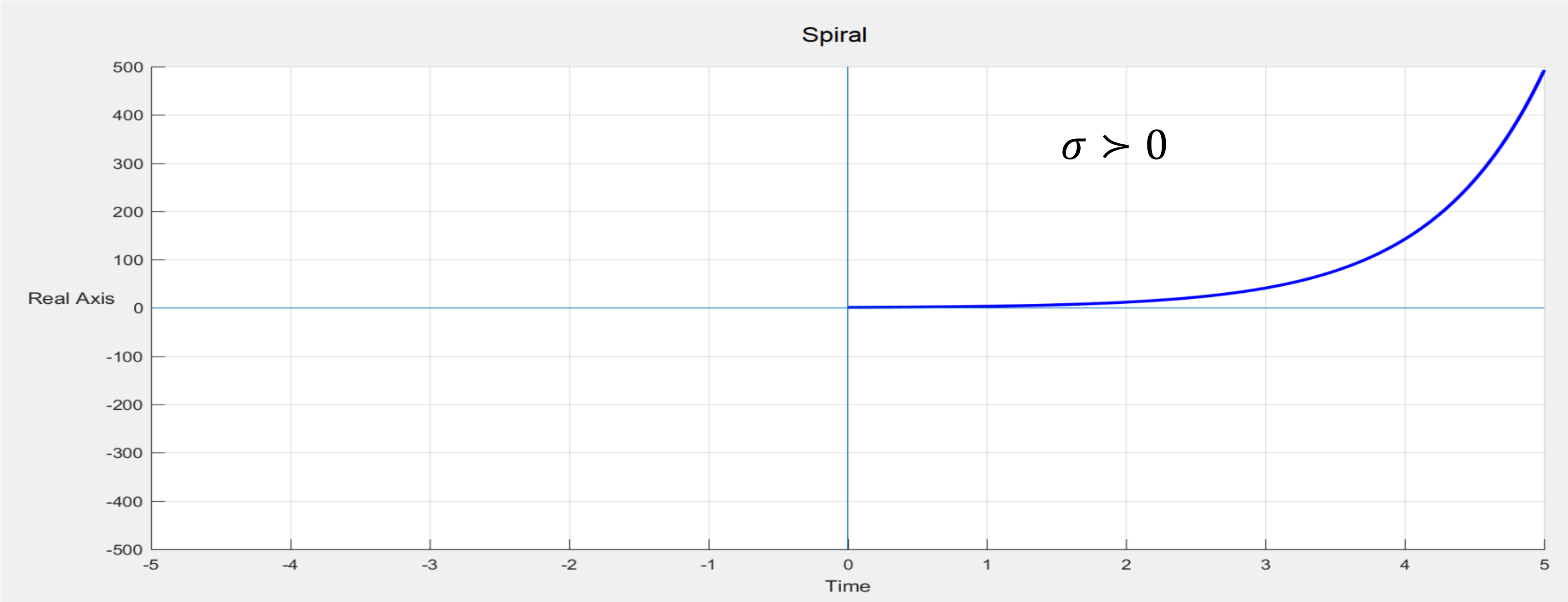


3-D View Plot

Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

$2.\ \ \omega = 0, s = \sigma, e^{st} = e^{\sigma t}$



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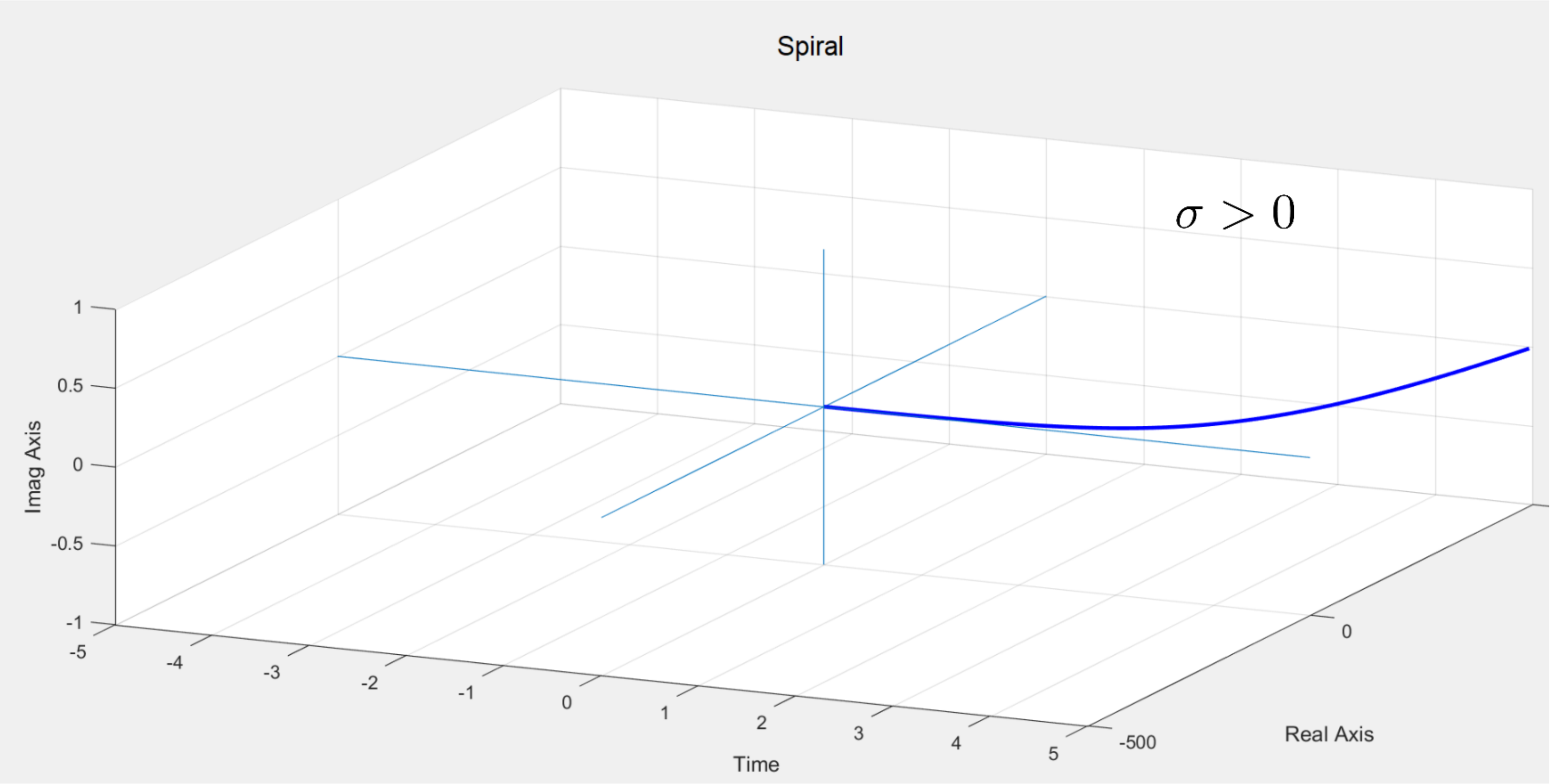

3-D View Plot

Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

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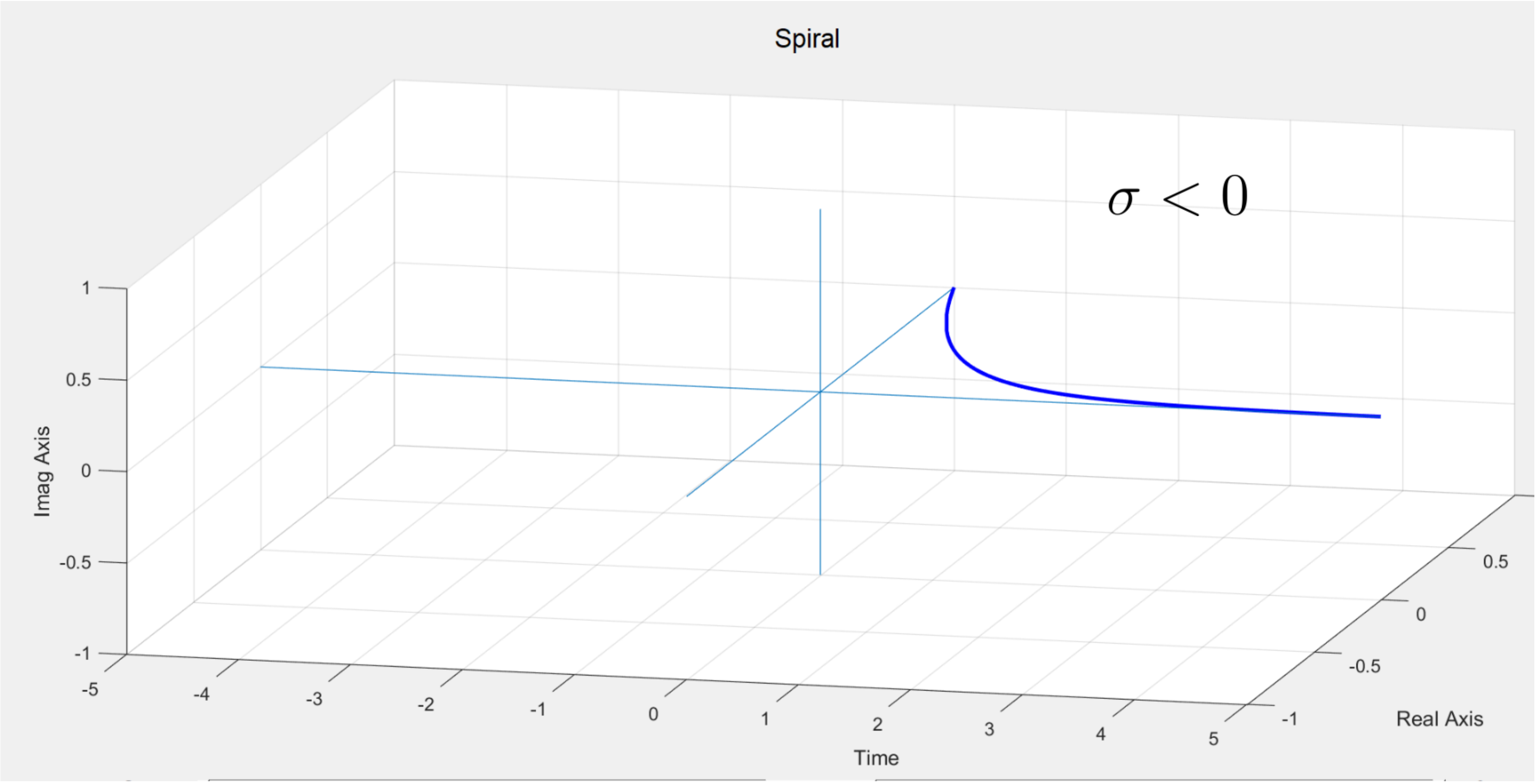
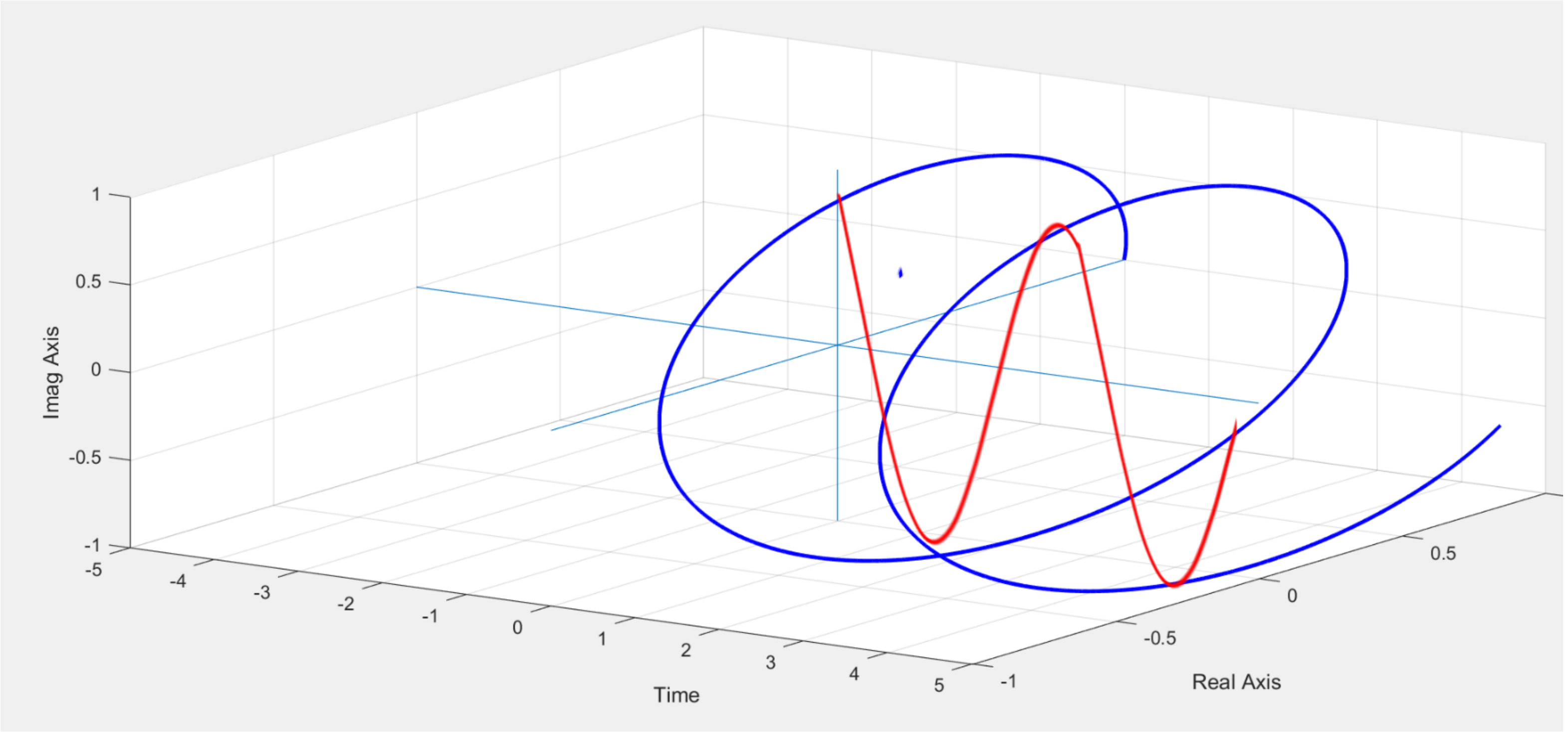


3-D View Plot

Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

3. $\sigma = 0, s = j\omega, e^{st} = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$



3-D View Plot

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$$e^{st} = e^{(\sigma + j\omega)t}$$

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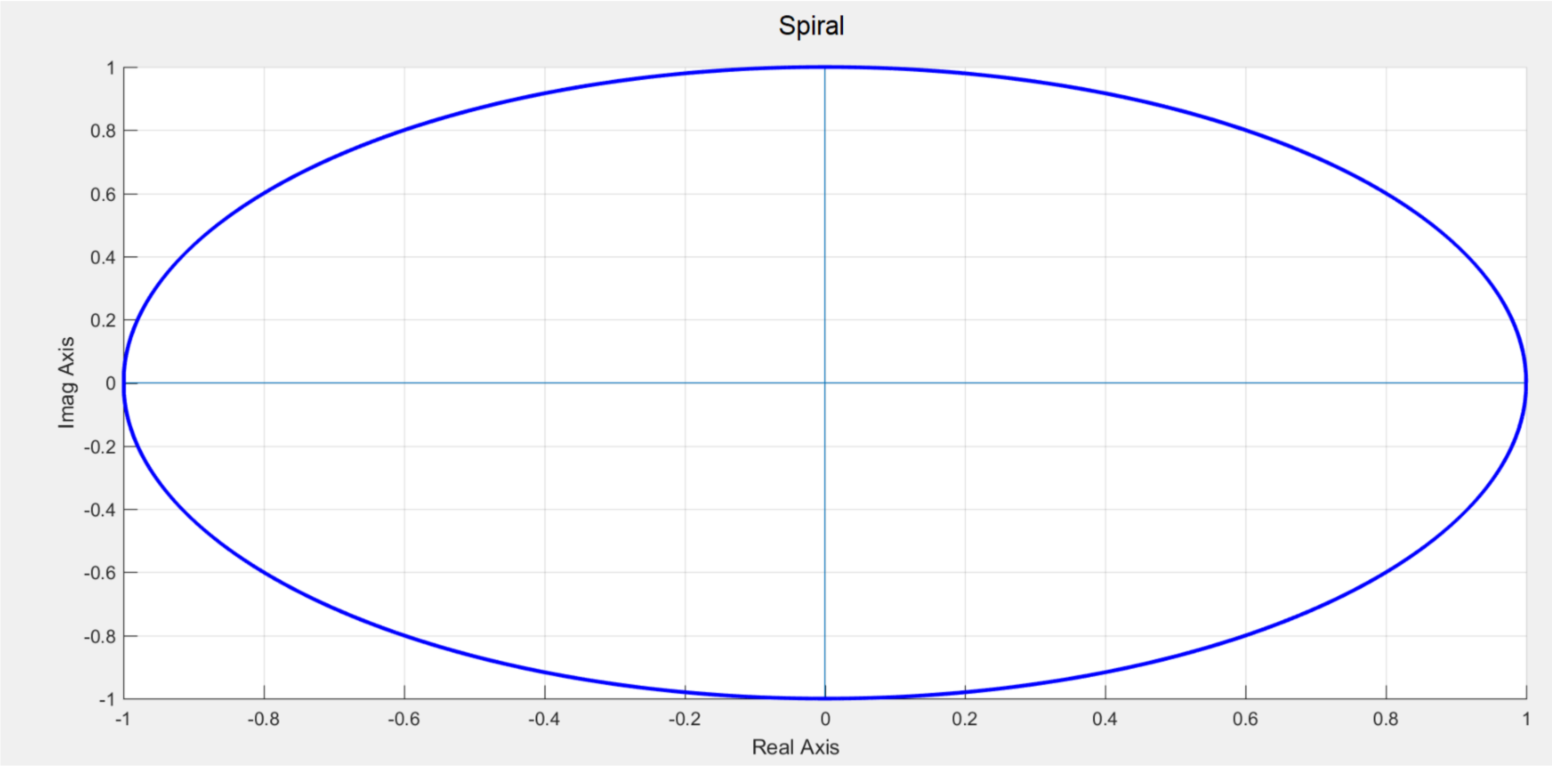


Front View Plot

Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

3. $\sigma = 0, s = j\omega, e^{st} = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

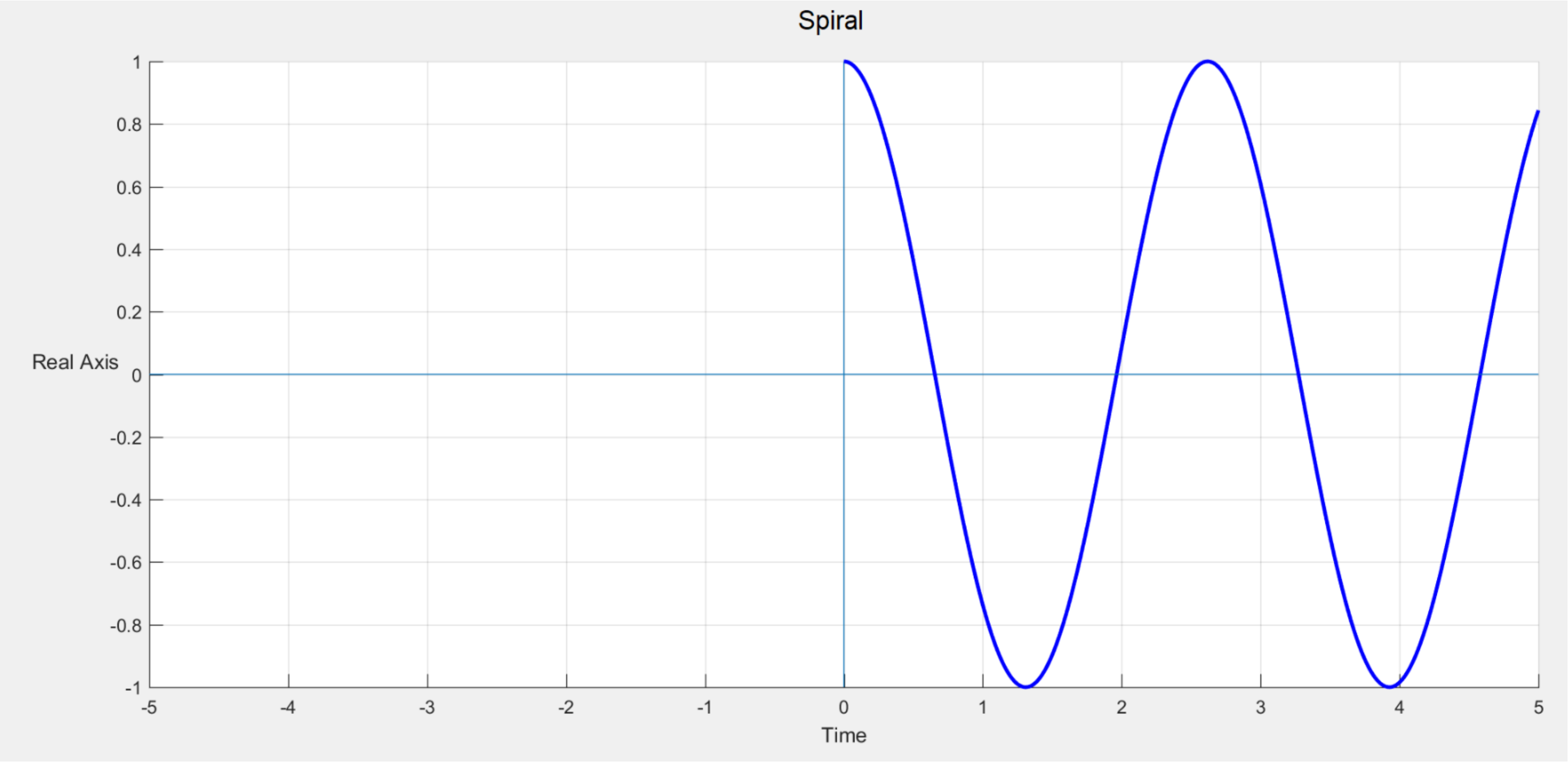
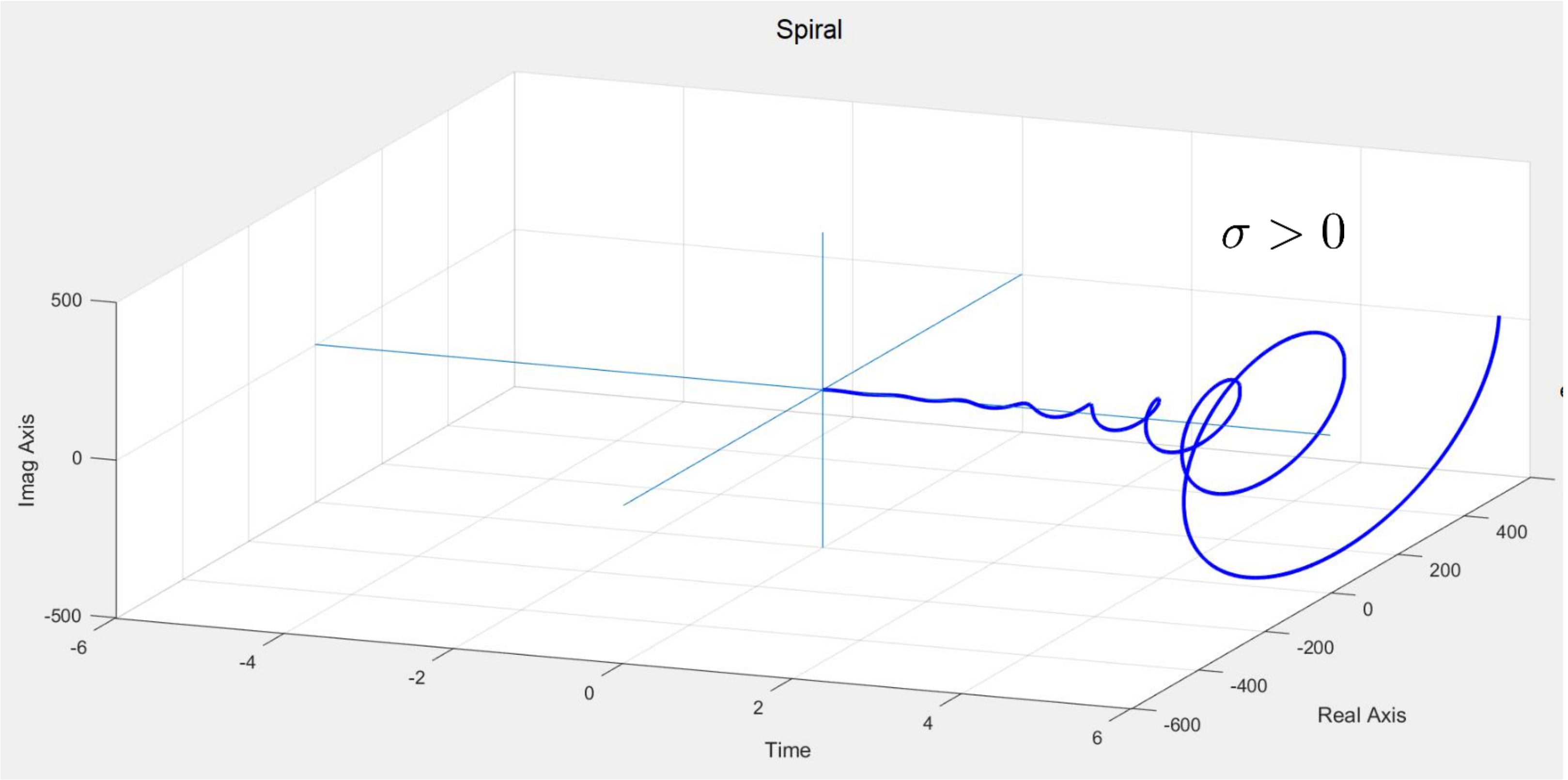


Illustration of e^{st} for different values of s

$4. s = \sigma + j\omega$

$e^{st} = e^{(\sigma + j\omega)t}$

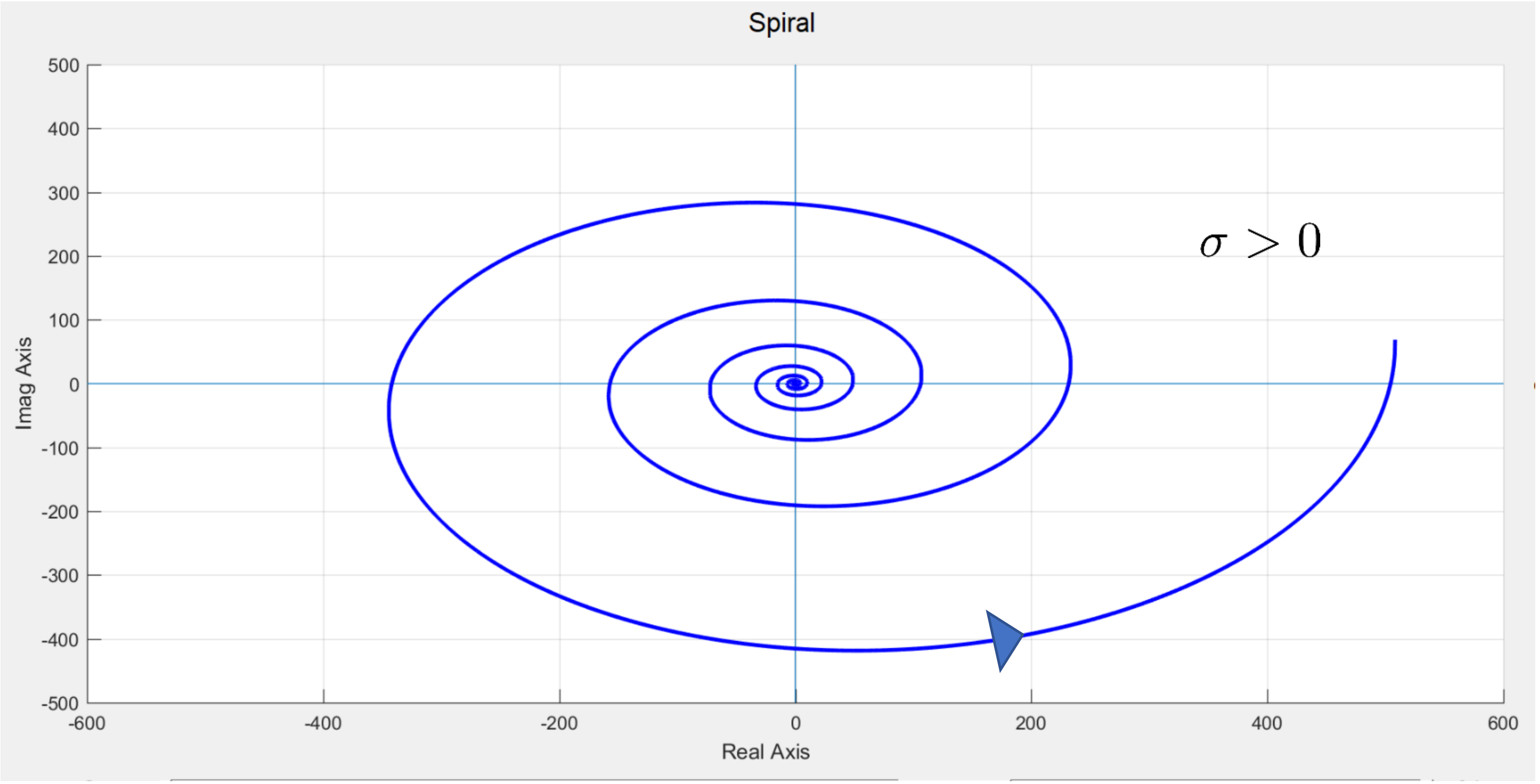


3-D View Plot

Illustration of e^{st} for different values of s

$4. s = \sigma + j\omega$

$e^{st} = e^{(\sigma + j\omega)t}$



Front View Plot

Illustration of e^{st} for different values of s

$4. s = \sigma + j\omega$

$$e^{st} = e^{(\sigma + j\omega)t}$$

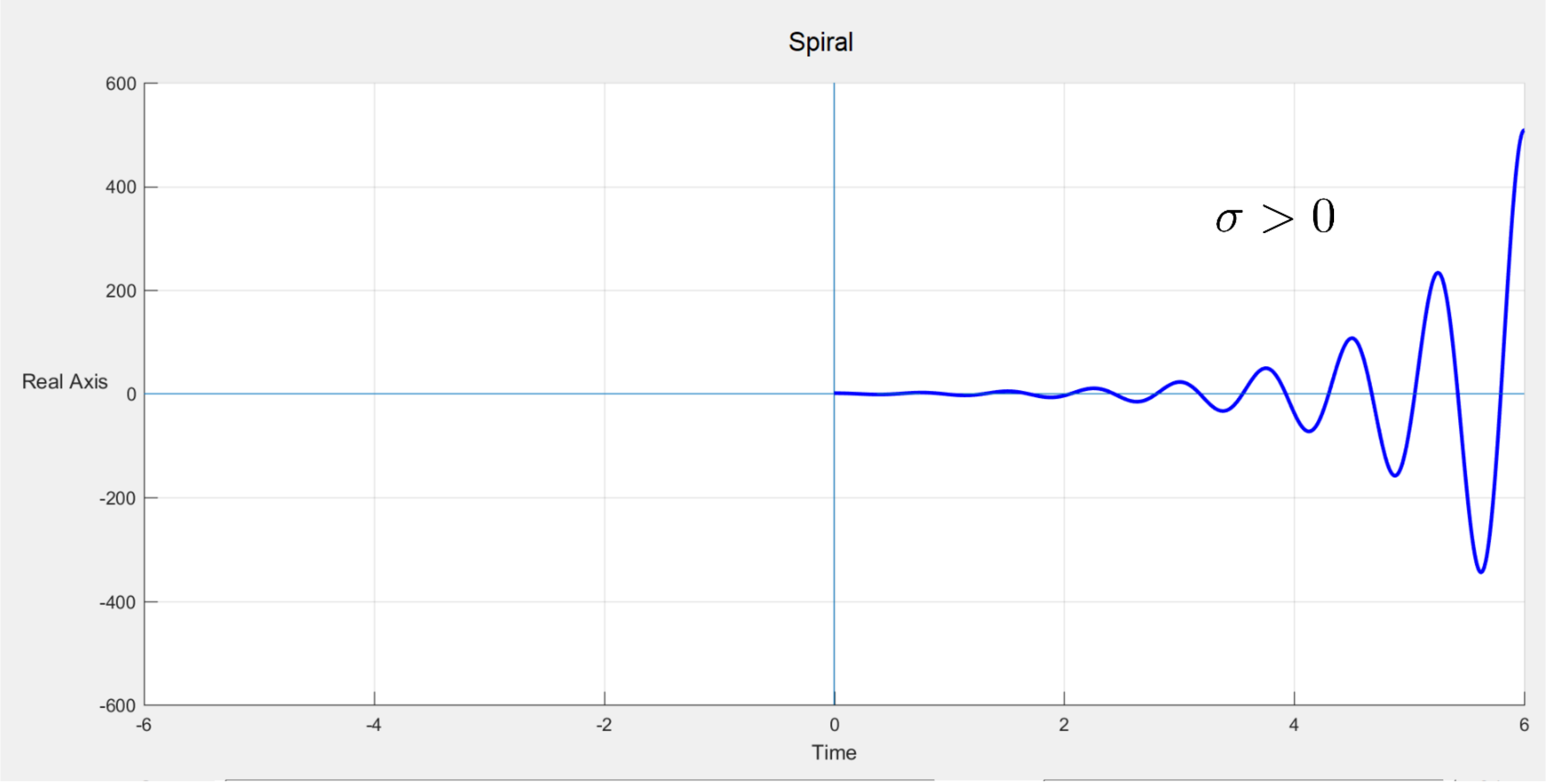
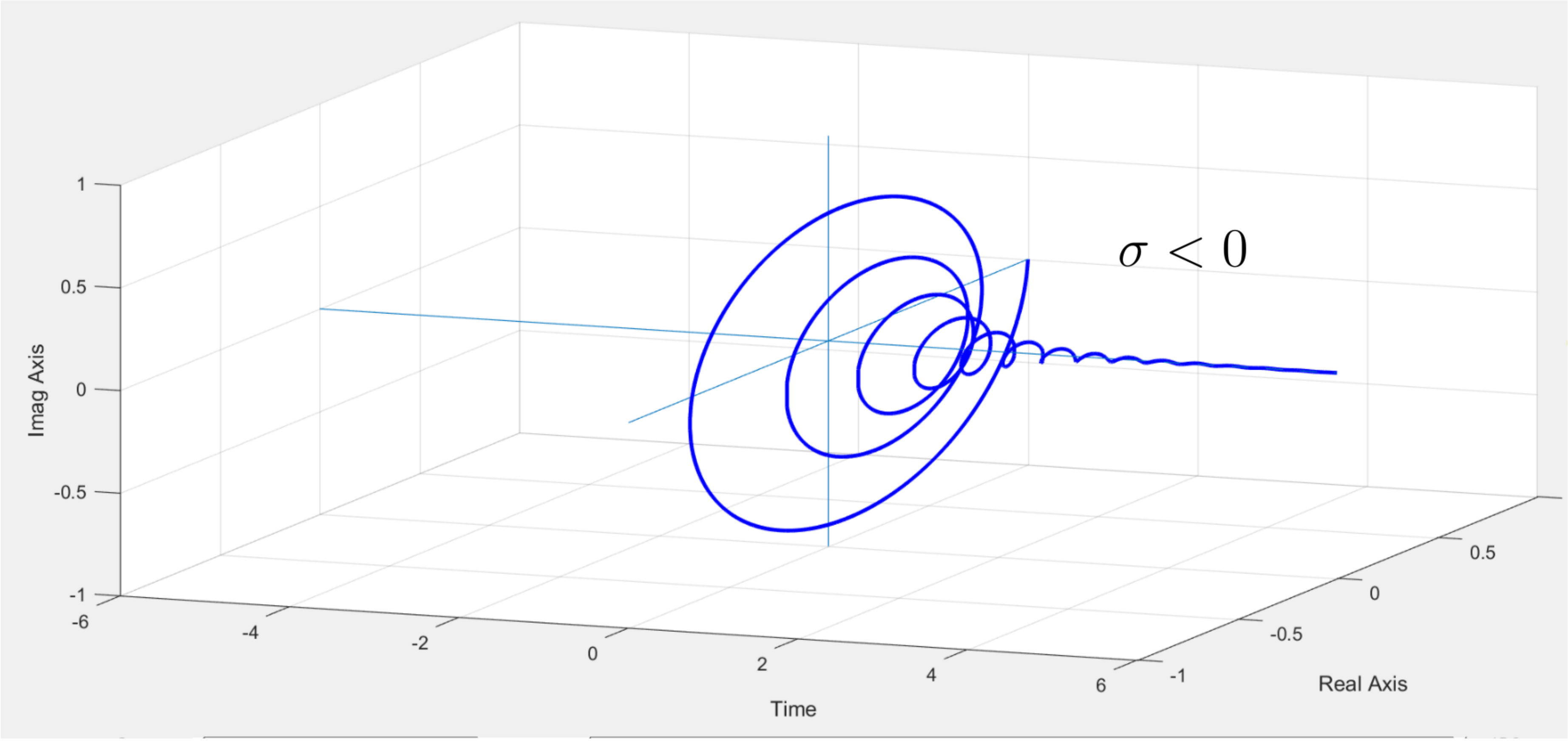


Illustration of e^{st} for different values of s

$4. s = \sigma + j\omega$ $e^{st} = e^{(\sigma + j\omega)t}$



3-D View Plot

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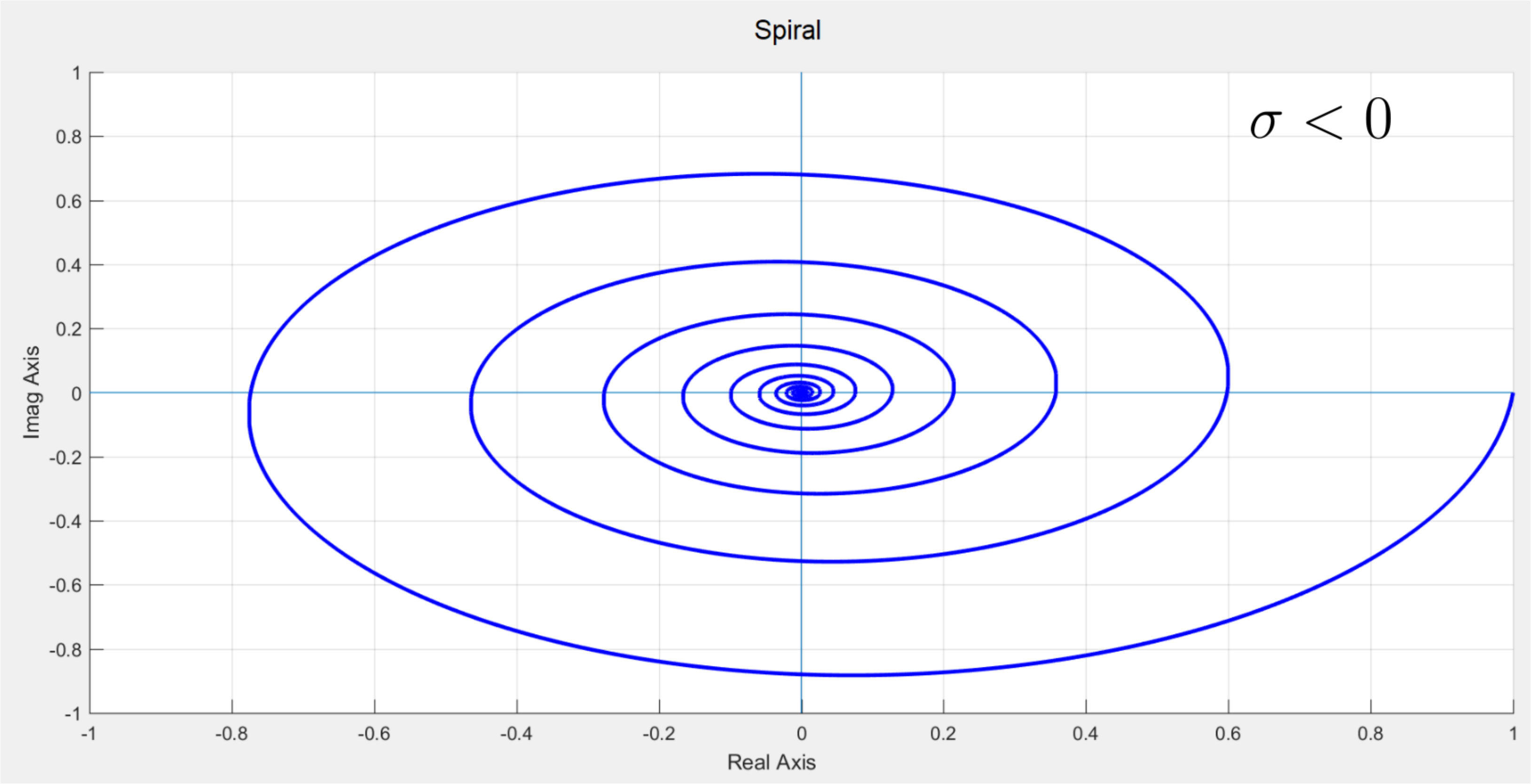
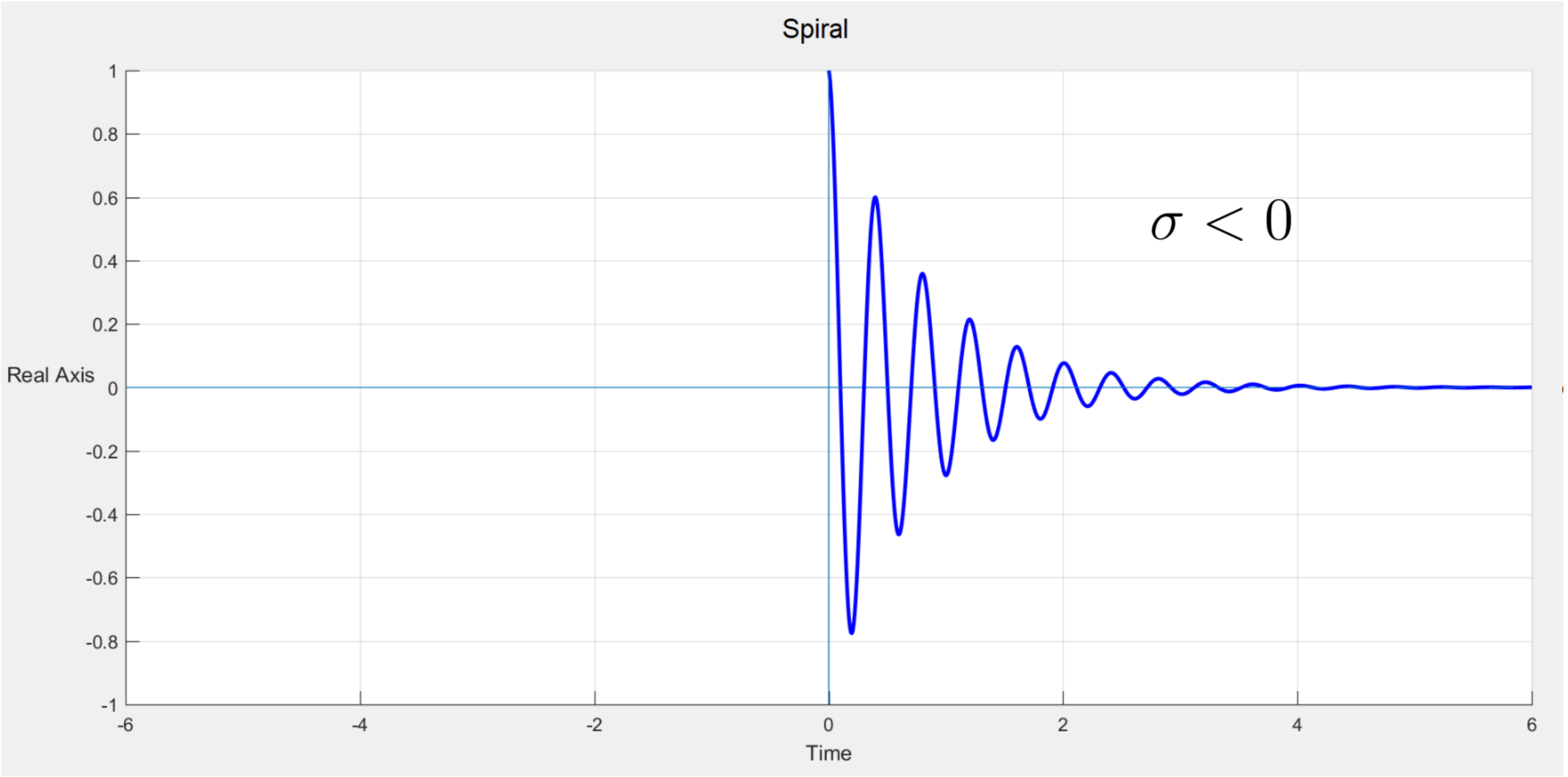


Front View Plot

Illustration of e^{st} for different values of *s*

$4.s = \sigma + j\omega$ $e^{st} = e^{(\sigma + j\omega)t}$

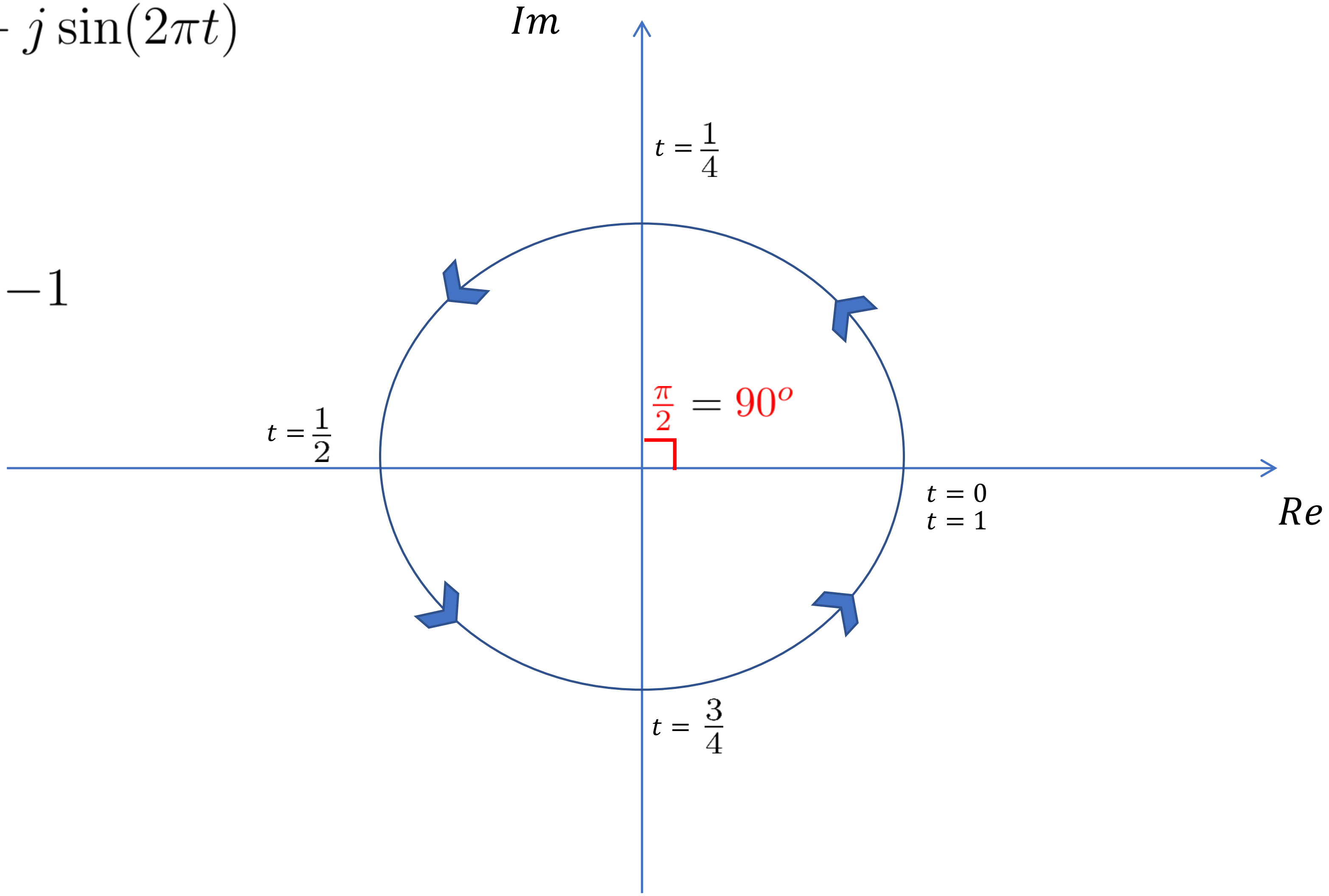


Exponential Signals:

Example: Consider the pure imaginary example of complex number: $e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$

For :

- $t = 0 \rightarrow e^{j2\pi(0)} = 1$
- $t = \frac{1}{2} \rightarrow e^{j\frac{2\pi}{2}} = e^{j\pi} = -1$
- $t = 1 \rightarrow e^{j2\pi} = 1$



Exponential Signals:

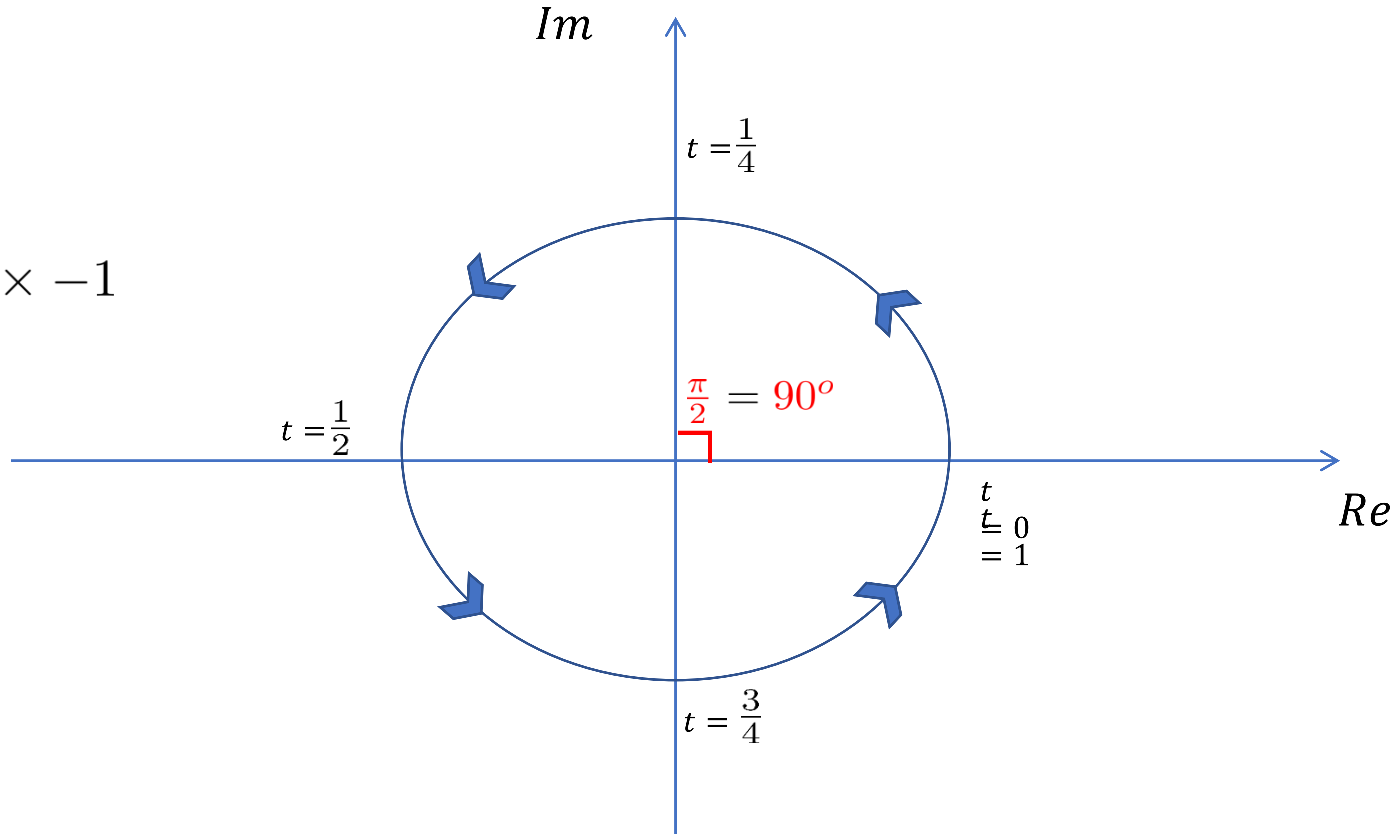
General Case: $e^{\sigma t}.e^{j\omega t} = e^{(\sigma + j\omega)t}$

σ *indicate the decay or expansion,* ω *indicate the speed of rotation*

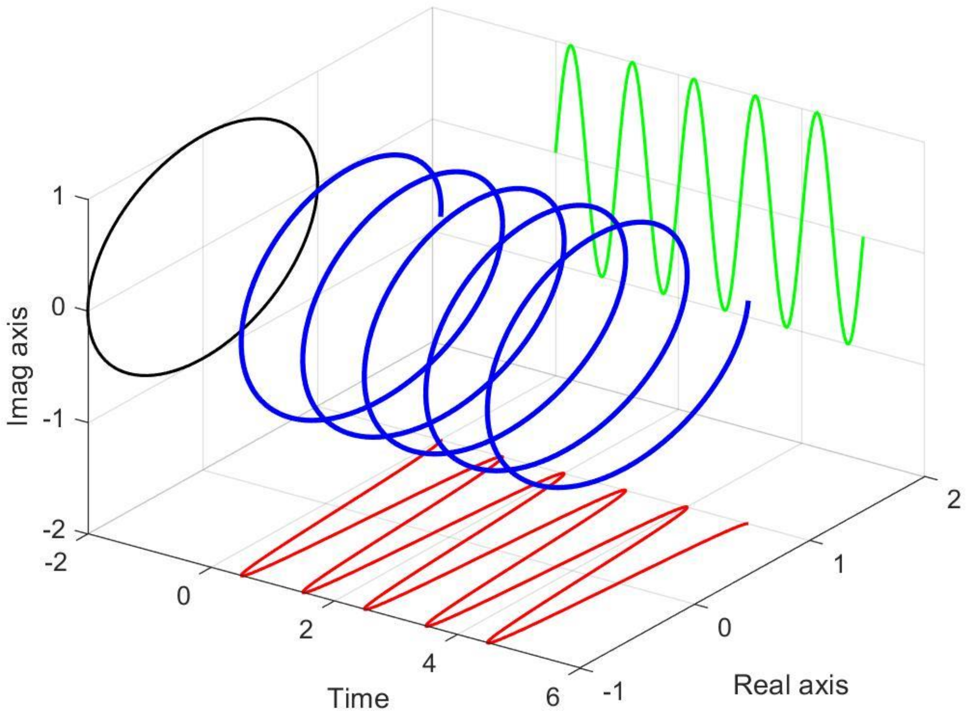
$$e^{-\frac{1}{2}t}e^{j2\pi t} = e^{-\frac{1}{2}t}(\cos(2\pi t) + j\sin(2\pi t))$$

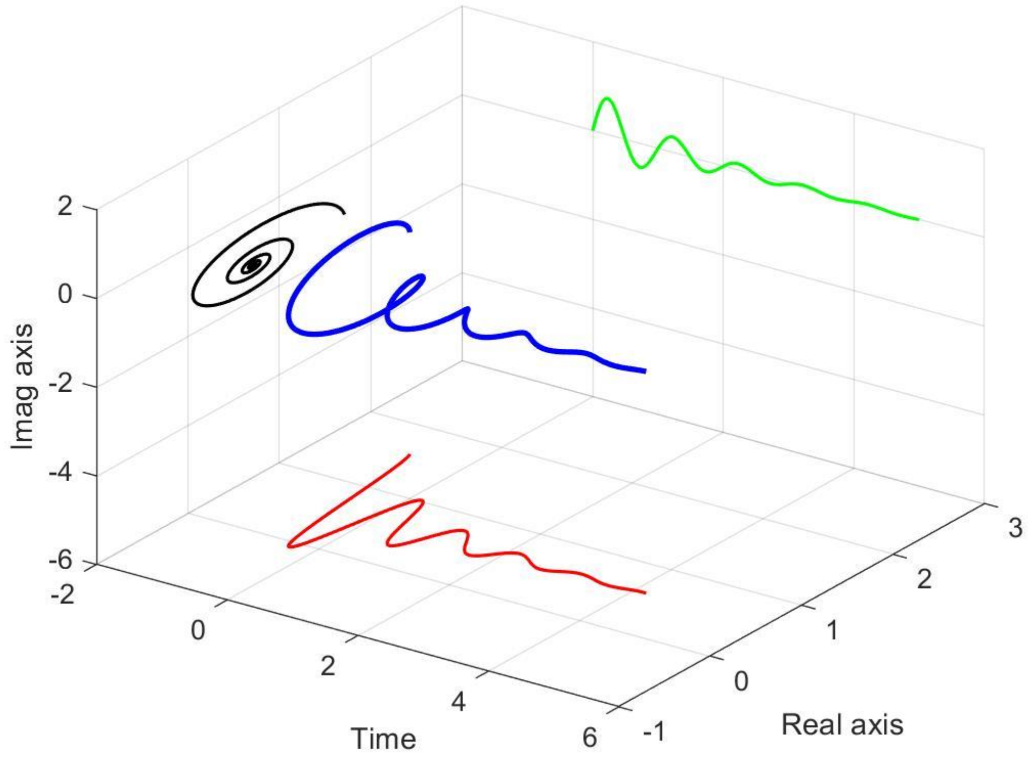
For :

- $t = 0 \rightarrow e^{0}e^{j2\pi(0)} = 1$
- $t = \frac{1}{2} \rightarrow e^{-\frac{1}{2}\times\frac{1}{2}}e^{j\frac{2\pi}{2}} = e^{-\frac{1}{4}}e^{j\pi} = e^{-\frac{1}{4}} \times -1$
- $t = 1 \rightarrow e^{-\frac{1}{2}}e^{j2\pi} = e^{-\frac{1}{2}}$



Exponential Signals





$$e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$$

$$e^{-.85t}e^{j2\pi t} = e^{-.85t}\cos(2\pi t) + je^{-.85t}\sin(2\pi t)$$