

Assignment 5

- Let X be an exponential random variable with rate λ .
 - Use the definition of conditional expectation to determine $E[X|X < c]$.
 - Now determine $E[X|X < c]$ by using the following identity:

$$E[X] = E[X|X < c]P[X < c] + E[X|X > c]P[X > c]$$

- Considering a post office with two clerks. Three people, A , B and C , enter simultaneously. A and B go directly to the clerks, and C waits until either A or B leaves before he begins service. What is the probability that A is still in the post office after the other two have left when
 - the service time for each clerk is exactly (non random) ten minutes?
 - the service times are i with probability $1/3$, $i = 1, 2, 3$?
 - the service times are exponential with mean $1/\mu$?
- Machine 1 is currently working. Machine 2 will be put in use at a time t from now. If the lifetime of machine i is exponential with rate λ_i , $i = 1, 2$, what is the probability that machine 1 is the first machine to fail?
- Two individuals, A and B , both require kidney transplants. If she does not receive a new kidney, then A will die after an exponential time with rate μ_A , and B after an exponential time with rate μ_B . New kidneys arrive in accordance with a Poisson process having rate λ . It has been decided that the first kidney will go to A (or to B if B is alive and A is not at that time) and the next one to B (if still living).
 - What is the probability that A obtains a new kidney?
 - What is the probability that B obtains a new kidney?
- The number of hours between successive train arrivals at the station is uniformly distributed on $(0, 1)$. Passengers arrive according to a Poisson process with rate 7 per hour. Suppose a train has just left the station. let X denote the number of people that get on the next train. Find
 - $E[X]$?

- (b) $\text{Var}(X)$?
6. A cable car starts off with n riders. The times between successive stops of the car are independent exponential random variables with rate λ . At each stop one rider gets off. This takes no time, and no additional riders get on. After a rider gets off the car, he or she walks home. Independently of all else, the walk takes an exponential time with rate μ .
- (a) What is the distribution of the time at which the last rider departs the car?
- (b) Suppose the last rider departs the car at time t . What is the probability that all the other riders are home at that time?
7. **Textbook 12.3.3** A wireless packet communications channel suffers from clustered errors. That is, whenever a packet has an error, the next packet will have an error with probability 0.9. When a packet is error-free, the next packet is error-free with probability 0.99. In steady-state, what is the probability that a packet has an error?
8. **Textbook 12.5.6** A very busy bank has two drive-thru teller windows in series served by a single line. When there is a backlog of waiting cars, two cars begin service simultaneously. The front customer can leave if she completes service before the rear customer. However, if the rear customer finishes first, he cannot leave until the front customer finishes. Consequently, the teller at each window will sometimes be idle if their customer completes service before the customer at the other window. Assume there is an infinite backlog of waiting cars and that service requirements of the cars (measured in seconds) are geometric random variables with a mean of 120 seconds. Draw a Markov chain that describes whether each teller is busy. What is the stationary probability that both tellers are busy?
9. **Textbook 10.3.4** Let $X(t) = e^{-(t-T)}u(t-T)$ be an exponential pulse with a random delay T . The delay T has a pdf $f_T(t)$. Find the pdf of $X(t)$