Assignment 2

1. We place at random n points in the interval (0,1) and we denote by random variables X and Y the distance from the origin to the first and the last points respectively. Find $F_X(x), F_Y(y)$ and $F_{X,Y}(x,y)$.

2. A random variable X has the density function

$$f_X(x) = \begin{cases} \frac{1}{2}\exp\left(-\frac{x}{2}\right) & x \geq 0 \\ 0 & \text{o.w.} \end{cases} \tag{1}$$

   Define events $A = \{1 < X \leq 3\}$, $B = \{X \leq 2.5\}$, and $C = A \cap B$. Find the probabilities of events A, B, and C.

3. Suppose height to the bottom of clouds is a Gaussian R.V. X for which $\mu = 4000$m, and σ=1000m. A person bets that cloud height tomorrow will fall in the set $A = \{1000m < X \leq 3300m\}$ while a second person bets that height will be satisfied by $B = \{2000m < X \leq 4200m\}$. A third person bets they are both correct. Find the probabilities that each person will win the bet.

4. A random variable X is known to be Poisson with $\lambda = 4$.

   (a) Plot the density and distribution functions for this random variable.

   (b) What is the probability of the events $\{0 \leq X \leq 5\}$?

5. A random variable X has a probability density

$$f_X(x) = \begin{cases} \frac{\pi}{16}\cos(\frac{\pi x}{8}) & -4 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

   Find: (a) its mean value $\bar{X}$, (b) its second moment $\bar{X^2}$, and (c) its variance.

6. A random variable has a probability density

$$f_X(x) = \begin{cases} \frac{5}{4}(1 - x^4) & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

   Find: (a) $E[X]$, (b) $E[4X + 2]$, and (c) $E[X^2]$.

7. Suppose a coin having probability 0.7 of coming up heads is tossed three times. Let X denote the number of heads that appear in the three tosses. Determine the probability mass function of X .

8. If the distribution function of F is given by

$$F(b) = \begin{cases} 0, & b < 0 \\ 1/2, & 0 \leq b < 1 \\ 3/5, & 1 \leq b < 2 \\ 4/5, & 2 \leq b < 3 \\ 9/10, & 3 \leq b < 3.5 \\ 1, & b \geq 3.5 \end{cases} \tag{4}$$

calculate and sketch the probability mass function of X.

9. On a multiple-choice exam with three possible answers for each of the five questions, what is the probability that a student would get four or more correct answers just by guessing?

10. Let X be a Poisson random variable with parameter λ. Show that $P(X = i)$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ. **Hints:** consider $P(X = i)/P(X = i - 1)$.

11. Let c be a constant. Show that

    (a) $\text{Var}(cX) = c^2\text{Var}(X)$.

    (b) $\text{Var}(c + X) = \text{Var}(X)$.

12. Suppose that X takes on each of the values 1,2,and 3 with probability 1/3. What is the moment generating function? Derive $E[X]$, $E[X^2]$, and $E[X^3]$ by differentiating the moment generating function and then compare the obtained result with a direct derivation of these moments.