Assignment 1

1. If the events A and B satisfy $AB = \phi$, then show that

$$P(A) \leq P(B^c). \tag{1}$$

2. If the events A and B have $P(A) = P(B) = P(AB)$, then show that

$$P(AB^c \cup BA^c) = 0. \tag{2}$$

3. If the events A and B have $P(A) = P(B) = 1$, then show that

$$P(AB) = 1. \tag{3}$$

4. If events A and B have $A \subset B$ with $P(A) = 1/4$ and $P(B) = 1/3$ then find $P(A|B)$ and $P(B|A)$.

5. Box A contains 1 white ball and 999 red balls while box B contains 1 red ball and 999 white balls. A ball is chosen at random from a randomly selected box and found to be red. What is the probability that it came from box A?

6. Suppose bin A contains 1000 parts of which 10% are defective while bin B contains 2000 parts of which 5% are defective. Two parts are picked at random from one bin selected at random.

   (a) Find the probability that both parts are defective.

   (b) If both parts are defective, find the probability that they came from bin A.

7. A prize is hidden behind one of three doors and you randomly choose one door in a contest to attempt to win the prize. Before checking your chosen door, the person running the contest opens one of the remaining two doors to show no prize there. Show that, without any further knowledge, you are more likely to win the prize if you change your choice from the door you originally picked to the other remaining closed door.

8. The random variable X is N(0,4), (with mean=0, var=4), find (a) $P\{1 \leq X \leq 2\}$; (b) $P\{1 \leq X \leq 2 \, |X \geq 1\}$.

9. A fair coin is tossed three times and the random variable X equals the total number of heads. Find and sketch $F_X(x)$ and $f_X(x)$.

10. The arrival time of a professor to his office is a continuous random variable uniformly distributed over the hour between 8 am and 9 am. Define the events:

$$\begin{aligned} A &= \{\text{ The Prof. has not arrived by 8:30 am}\} \\ B &= \{\text{ The prof. will arrive by 8:31 am }\}. \end{aligned} \tag{4}$$

Find

(a) $P(B|A)$

(b) $P(A|B)$.

11. Let

$$f_X(x) = \begin{cases} ce^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

(a) Find c;

(b) Let $a > 0, x > 0$, find $P(X \geq x + a)$;

(c) Let $a > 0, x > 0$, find $P(X \geq x + a | X \geq a)$;